

## Digital Processing of Speech and Image Signals

### 1. Exercise

Submission of the solutions: 30. 10. 2006 at the beginning of the lecture

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#### General Information:

- The exercises should be treated in groups of 3 students.
- Requirement for the issuing of certificates (Schein/Leistungsnachweis) will be that at least 50% of the exercise points are obtained.
- Solutions to problems without points will be presented in the exercise lesson.
- For the lecture there exists a web site, which gives information on current dates and possibly important announcements. There also the exercise sheets as well as other documents needed for the solution of the exercises will be made available. You will find the web site at:

[http://www-i6.informatik.rwth-aachen.de/web/Teaching/Lectures/WS06\\_07/Signal/](http://www-i6.informatik.rwth-aachen.de/web/Teaching/Lectures/WS06_07/Signal/)

#### Task 1.1

Check, if the following transformations  $y(t) = S\{x(t)\}$  describe linear, time invariant (LTI) systems.

- (a)  $y(t) = \frac{d}{dt}x(t)$  (1 P.)
- (b)  $y(t) = x(t - \tau)$  (1 P.)
- (c)  $y(t) = 1 + x(t)$  (1 P.)
- (d)  $y(t) = m(t) \cdot x(t)$  (1 P.)
- (e)  $y(t) = t^2 \cdot x(t)$
- (f)  $y(t) = |x(t)|$

#### Task 1.2

Calculate the convolution integral  $y(t) = h(t) * x(t)$  and sketch  $y(t)$ .

- $x(t) = \text{rect}\left(\frac{t}{T_1}\right)$ ,  $h(t) = \text{rect}\left(\frac{t}{T_2}\right)$ ,  $T_1 < T_2$  (2 P.)
- $x(t) = a \cdot \text{rect}\left(\frac{t - T_0/2}{T_0}\right)$ ,  $h(t) = \frac{1}{T} \epsilon(t) e^{-\frac{t}{T}}$ ,

Hint:  $\text{rect}(t/T) = 1$  for  $|t| \leq T/2$ , otherwise 0.  
 $\epsilon(t) = 1$  for  $t \geq 0$ , otherwise 0.

### Task 1.3

Calculate the Fourier integral for the signal  $h(t)$ .

(a)  $h(t) = \epsilon(t)e^{-at} \cos(\beta t)$  (2 P.)

(b)  $h(t) = \text{rect}\left(\frac{t-t_0}{T}\right)$

### Task 1.4

Assume an integrator, i.e. a system that carries out the transformation  $y(t) = \int_{-\infty}^t x(\tau) d\tau$ .

(a) Show that the integrator is a linear time invariant (LTI) system. (1 P.)

(b) Represent the integrator by a convolution integral. What is the impulse response  $h_1(t)$  of the integrator? (1 P.)

(c) Represent a system whose impulse response  $h_2(t)$  is a rectangle function of length  $T$  by using the integrator. Calculate the corresponding transformation equation. (1 P.)

(d) Give a descriptive interpretation of the transformation equation. What kind of system is this? (1 P.)



Jean Baptiste Joseph Fourier (1768 - 1830) was a French mathematician and physicist. In 1822 he published his *Théorie analytique de la chaleur* (*Analytical Theory of Heat*), in which he bases his reasoning on Newton's law of cooling, namely, that the flow of heat between two adjacent molecules is proportional to the extremely small difference of their temperatures. In this work he claims that any function of a variable, whether continuous or discontinuous, can be expanded in a series of sines of multiples of the variable. Though this result is not correct, Fourier's observation that some discontinuous functions are the sum of infinite series was a breakthrough. "The differential equations of the propagation of heat express the most general conditions, and reduce the physical questions to problems of pure analysis, and this is the proper object of theory."

Source: <http://www.wikipedia.com>