

Digital Processing of Speech and Image Signals

2. Exercise

Submission of the solutions: 06. 11. 2006 at the beginning of the lecture

Task 2.1

Consider a Gaussian impulse $s_0(t) = s(t - t_0) = e^{-\pi(t-t_0)^2}$ for given t_0 .

- (a) Calculate the Fourier integral for this signal. (1 P.)

Hint:

$$\int_0^{\infty} e^{-a^2 x^2} \cos(bx) dx = \frac{\sqrt{\pi}}{2a} e^{-b^2/(4a^2)} \text{ für } a > 0$$

- (b) Calculate the convolution integral for two Gaussian impulses

$$s_0(t) = s(t - t_0) \text{ and } s_1(t) = s(t - t_1).$$

(1 P.)

Hint:

$$\int e^{-a(\mu/2-x)^2} dx = -\frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(\frac{1}{2}\sqrt{a}(\mu - 2x)\right)$$

$$\text{with } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad \operatorname{erf}(\infty) = 1, \operatorname{erf}(-x) = -\operatorname{erf}(x)$$

- (c) Calculate the convolution integral for n Gaussian impulses $s_0(t) \dots s_{n-1}(t)$.

Solve this problem using the convolution theorem. (1 P.)

- (d) Sketch the solution of exercise c) for $n = 1, 2, 3$ and $t_i = 0$ for all i . (1 P.)

Task 2.2 Calculate the Fourier integral for the signal $h(t)$. (3 P.)

$$h(t) = \begin{cases} 0 & |t| \geq 1 \\ -1 & 0 \leq t < 1 \\ 1 & -1 < t < 0 \end{cases}$$

Task 2.3

(a) Given a function $g(t)$ and its Fourier transform $G(\omega) = F\{g(t)\}$, what is the Fourier integral $F\{G(t)\}$? (1 P.)

(b) Calculate the Fourier integrals for the following functions

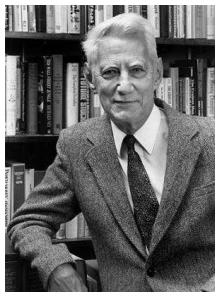
i) $s(t) = \text{si}(\alpha t) = \frac{\sin(\alpha t)}{\alpha t}$

ii) $s(t) = \text{si}^2(\alpha t) = \frac{\sin^2(\alpha t)}{(\alpha t)^2}$ (1 P.)

iii) $s(t) = \frac{\alpha}{\alpha^2 + t^2}$ (1 P.)

Task 2.4 Sketch the function $h(t)$, the spectrum $H(\omega)$, the autocorrelation function $R(t)$, and the power spectrum $|H(\omega)|^2$ for the signal (4 P.)

$$h(t) = \text{rect}(t).$$



Claude Elwood Shannon (1916 - 2001), an American electrical engineer and mathematician, has been called *the father of information theory*, and was the founder of practical digital circuit design theory. His most famous work *A Mathematical Theory of Communication* was published in 1948. Shannon developed information entropy as a measure for the uncertainty in a message while essentially inventing what became known as the dominant form of information theory. Another notable paper published in 1949 is *Communication Theory of Secrecy Systems*, a major contribution to the development of a mathematical theory of cryptography. He is also credited with the introduction of Sampling Theory.

Explaining why he named his uncertainty function “entropy”, he said: “My greatest concern was what to call it. I thought of calling it ‘information,’ but the word was overly used, so I decided to call it ‘uncertainty.’ When I discussed it with John von Neumann, he had a better idea. Von Neumann told me, ‘You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, no one really knows what entropy really is, so in a debate you will always have the advantage.’”

Source: <http://www.wikipedia.com>