Chair of Computer Science 6
RWTH Aachen University
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Digital Processing of Speech and Image Signals

6. Exercise

Submission of the solutions: 04. 12. 2006 at the beginning of the lecture

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For the calculation, implementation and plotting of the following exercises you will need a mathematical toolkit. The public domain toolkit Octave is available at `http://www.octave.org`.

Task 6.1

A real time-continuous signal $s(t)$ is band-limited to $f_B = 5$kHz. The signal is sampled at a rate of 10000 samples per second. $X[k]$ values are calculated on the samples by DFT with a window length $N = 512$.

(a) Which frequencies do the following DFT coefficients belong to:
$X[0]$, $X[150]$, $X[256]$, $X[500]$? (1 P.)

(b) Can you state anything about the frequency $f = 4$kHz? (1 P.)

(c) Let us change the DFT length to $N = 2048$. Which frequency does $X[150]$ belong to? (1 P.)

Task 6.2 The signal from Task 1 is sampled at the frequency f_S determined according to the sampling theorem. We apply a DFT of length N where

- N is a power of two and
- the distance between the position of two DFT coefficients is maximum 5Hz.

Determine the minimal value for N. What range of sampling frequencies ($f_{min} \leq f_S \leq f_{max}$) do the constraints allow at the chosen value of N.

(2 P.)

Task 6.3 Given the signal

$$s(t) = A_0 \cos(\omega_0 t) + A_1 \cos(\omega_1 t).$$

(a) Give a formula for the sampled discrete signal $s[n]$. (1 P.)

(b) Give a formula for the signal $v[n]$ which is generated by applying a window function $w[n]$ to $s[n]$. Determine the corresponding Fourier transform $V(e^{i\omega})$. (1 P.)

(c) Implement the Discrete Fourier Transform in a mathematical toolkit. The function has to support arbitrary signal length resp. arbitrary window length: `spectrum = dft(signal)`. (1 P.)

Following parameters are given further on in this exercise:

$$1/T_S = 10\text{kHz} \quad A_0 = 1 \quad A_1 = 0.75 \quad \omega_0 = \frac{2\pi}{14}10\text{kHz} \quad \omega_1 = \frac{4\pi}{15}10\text{kHz}$$

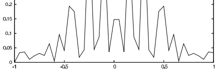
Use a rectangular window function of the length 64.

(d) Plot $v[n]$. (1 P.)

(e) Perform the DFT with length $N = 64$ and plot the real, the imaginary part and the amplitude of the spectrum. (1 P.)

(f) Interpret the curve of $|V[n]|$. (1 P.)

(g) Perform the DFT with length $N = 1024$ and plot the amplitude spectrum of $V[n]$ again. (Hint: use zero-padding.) (1 P.)



Arthur Schuster (1851 - 1934) was a versatile physicist known for his work in spectroscopy, electrochemistry, optics, X-radiography and the application of harmonic analysis to physics. He contributed enormously to making the University of Manchester a centre for the study of physics.
Schuster is perhaps most widely remembered for his periodogram analysis, a technique which was long the main practical tool for identifying statistically important frequencies present in a time series of observations. He first used this form of harmonic analysis in 1897 to disprove C. G. Knott's claim of periodicity in earthquake occurrences. He went on to apply the technique to analysing sunspot activity. (A. Schuster, *On the Investigation of Hidden Periodicities with Application to a Supposed Twenty-Six-Day Period of Meteorological Phenomena*, 1898)
Source: `http://www.wikipedia.com`