Features for Image Retrieval

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Overview

Introduction
State Of The Art
Features
Distance Measures
Performance Evaluation
Databases & Results
  – Features
  – Distance Measures
  – Databases
Improving text based image search
Conclusion & Perspective
Introduction

Motivation
Increasing amount of digitally available images
Methods for access to image databases are required

Aim
Development of a content-based image retrieval system
Focus on different features

Results
FIRE – Flexible Image Retrieval Engine
Capable of using many different features & distance measures
Quantitative results for different databases
## State Of The Art

<table>
<thead>
<tr>
<th></th>
<th>University</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMBA</td>
<td>University Freiburg</td>
<td>invariant feature histograms</td>
</tr>
<tr>
<td>GIFT</td>
<td>University Geneva</td>
<td>based on textual information retrieval</td>
</tr>
<tr>
<td>BlobWorld</td>
<td>University of California, Berkeley</td>
<td>region-based features</td>
</tr>
<tr>
<td>SIMPLIcity</td>
<td>Pennsylvania State University</td>
<td>preclassification, region-based features</td>
</tr>
<tr>
<td>QBIC</td>
<td>IBM Almaden</td>
<td>color histograms, texture histograms, shape moments</td>
</tr>
</tbody>
</table>

**Problem:** No quantitative results for comparison
Features

Color histograms

Invariant feature histograms, invariant feature vectors

Gabor features (& histograms)

Tamura texture features

Global texture descriptors

Local features (& histograms)

Region-based features, BlobWorld features

Pixel values

Fourier Mellin features

Image size
Invariant Feature Histograms

Theory:

\[ F(X) = \int_{g \in G} f(gX) \, dg \quad \text{invariant against any transformation from } G \]

Practice:

Consider rotations and translations

Choose \( f(X) = \sqrt{X(4, 0) \cdot X(0, 8)} \)

Replace integral by sums

Histogramization:

Replace one or more sums by histogramization:

\[ H_{F}(X) = \sum_{t_0=1, t_1=1}^{T_0,T_1} \frac{1}{R} \sum_{r=1}^{R} f\left( g_{t_0,t_1,\frac{2\pi r}{R}}X \right) \]
Distance Measures & Decision Rule

Histogram comparison measures
– Minkowski: Euclidean, $L_1$
– Histogram intersection
– Relative deviation
  Relative bin deviation
– $\chi^2$ distance
– Kullback-Leibler divergence, Jensen-Shannon divergence
– Bhattacharyya based distances
– Quadratic forms
– Earth movers distance
– Time warp distance

Image comparison measures
– Euclidean distance
– Tangent distance
– Image distortion model

Local feature comparison measures
– Direct transfer
– Image based

Region comparison measures
– Integrated region matching
– Quantized Hungarian matching

$$D(Q, X) := \sum_{i=1}^{I} w_i \cdot d_i(Q_i, X_i)$$

return $X_n$ with $n = \arg\min_{n'}\{D(Q, X_{n'})\}$
Performance Evaluation

Relevances known

\[ \text{Rank}_1, \text{Rank}, \]
\[ P(1), P(20), P(50), P(N_R), \]
\[ R(P = 0.5), R(100), \]
\[ PR\text{-graph}, PR\text{-area}, \]
\[ P(R = P) \]

\[ P = \frac{\#(\text{relevant})}{\#(\text{retrieved})} \]

\[ R = \frac{\#(\text{relevant})}{\#(\text{relevant in database})} \]

Relevances from annotation

Stemming of annotation to avoid problems with plurals, genitives...

Assume image to be relevant if same word in annotation

Problem: annotation ambiguous
## Correlation of performance measures

<table>
<thead>
<tr>
<th></th>
<th>P(1)</th>
<th>ER</th>
<th>P(50)</th>
<th>R(P=0.5)</th>
<th>R(100)</th>
<th>Rank1</th>
<th>Rank</th>
<th>P(R=P)</th>
<th>P(R-area)</th>
<th>P(R=0)</th>
<th>P(R=0.1)</th>
<th>P(R=0.5)</th>
<th>P(R=0.9)</th>
<th>P(R=1)</th>
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</thead>
<tbody>
<tr>
<td>P(1)</td>
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<tr>
<td>R(P=0.5)</td>
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<td>P(R=0.1)</td>
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<td>P(R=0.5)</td>
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<td>P(R=0.9)</td>
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<tr>
<td>P(R=1)</td>
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</tr>
</tbody>
</table>
Databases Part 1

WANG

1000 images, 10 classes

IRMA-1617

1617 images, 6 classes
## Using Different Features

### WANG

<table>
<thead>
<tr>
<th>Feature</th>
<th>ER[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>InvFeatHisto</td>
<td>15.9</td>
</tr>
<tr>
<td>Color histogram</td>
<td>17.9</td>
</tr>
<tr>
<td>Tamura histogram</td>
<td>31.0</td>
</tr>
<tr>
<td>LF(19×19) histogram 256</td>
<td>32.5</td>
</tr>
<tr>
<td>32×32</td>
<td>55.1</td>
</tr>
</tbody>
</table>

### IRMA-1617

<table>
<thead>
<tr>
<th>Feature</th>
<th>ER[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>32×X</td>
<td>6.7</td>
</tr>
<tr>
<td>LF(5×5) histogram 512</td>
<td>9.3</td>
</tr>
<tr>
<td>Tamura histogram</td>
<td>19.3</td>
</tr>
<tr>
<td>InvFeatHisto</td>
<td>22.6</td>
</tr>
<tr>
<td>Fourier Mellin feature</td>
<td>53.1</td>
</tr>
</tbody>
</table>

[Graphs showing precision and recall for different features for WANG and IRMA-1617]
Correlation Between Features: WANG

Graph from multi-dimensional scaling of $1 - |\text{cor}(\cdot, \cdot)|$. 
Correlation Between Features: IRMA

Graph from multi-dimensional scaling of $1 - |\text{cor}(\cdot, \cdot)|$. 
Using Different Distance Measures

<table>
<thead>
<tr>
<th>Distance measure</th>
<th>ER[%]</th>
<th>Distance measure</th>
<th>ER[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>JSD</td>
<td>15.9</td>
<td>$L_1$</td>
<td>8.3</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>16.5</td>
<td>$\chi^2$</td>
<td>9.1</td>
</tr>
<tr>
<td>$L_1$</td>
<td>18.4</td>
<td>JSD</td>
<td>9.3</td>
</tr>
<tr>
<td>Euclidean</td>
<td>28.3</td>
<td>Euclidean</td>
<td>14.2</td>
</tr>
</tbody>
</table>

![Graph comparing precision and recall for different distance measures](image-url)
Results for IRMA

**IRMA-1617**

<table>
<thead>
<tr>
<th>Method</th>
<th>ER[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>extended tangent distance</td>
<td>[KEYSERS00] 8.0</td>
</tr>
<tr>
<td>local features &amp; tangent distance</td>
<td>[KOELSCH03] 7.4</td>
</tr>
<tr>
<td>image distortion model</td>
<td>[GOLLAN03] 6.7</td>
</tr>
</tbody>
</table>

Feature selection (cross validation) 6.1

**IRMA-3879**

3879 images from 8 classes subdivided into training and test set

<table>
<thead>
<tr>
<th>training</th>
<th>L1O ER[%] on training data</th>
<th>ER[%] test data</th>
</tr>
</thead>
<tbody>
<tr>
<td>only image distortion model</td>
<td>7.5</td>
<td>7.2</td>
</tr>
<tr>
<td>feature weighting $w_i = 1.0$</td>
<td>9.2</td>
<td>9.1</td>
</tr>
<tr>
<td>$w_i \in {0, 1}$</td>
<td>7.5</td>
<td>7.3-9.1</td>
</tr>
<tr>
<td>$w_i \in {0, \ldots, 10}$</td>
<td>6.6</td>
<td>6.3-7.0</td>
</tr>
</tbody>
</table>
Results for WANG

3 different setups

WANG and UW database

1109 images, annotated

WANG and Corel subset

1000 images, 10 classes

WANG subsets

10 classes, 50 images each
## Results for WANG and UW database

<table>
<thead>
<tr>
<th>Training</th>
<th>ER[%] WANG</th>
<th>ER[%] UW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i = 1.0$</td>
<td>12.7</td>
<td>12.2</td>
</tr>
<tr>
<td>WANG $w_i \in {0, \ldots, 10}$</td>
<td>9.9</td>
<td>13.5</td>
</tr>
<tr>
<td>UW database $w_i \in {0, \ldots, 10}$</td>
<td>15.1</td>
<td>9.4</td>
</tr>
</tbody>
</table>

Training does not work

**Problem:** Databases too different
Results for WANG and Corel subset database

<table>
<thead>
<tr>
<th>Training on</th>
<th>ER[%] WANG</th>
<th>ER[%] Corel subset</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i = 1.0$</td>
<td>13.3</td>
<td>21.7</td>
</tr>
<tr>
<td>WANG $w_i \in {0, \ldots, 1}$</td>
<td>12.0</td>
<td>21.6</td>
</tr>
<tr>
<td>Corel subset $w_i \in {0, \ldots, 1}$</td>
<td>13.2</td>
<td>20.5</td>
</tr>
</tbody>
</table>

More similar databases

Training works
Results for WANG subsets

<table>
<thead>
<tr>
<th>Training on</th>
<th>ER[%]</th>
<th>WANG even</th>
<th>WANG odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i = 1.0$</td>
<td></td>
<td>14.4</td>
<td>17.4</td>
</tr>
<tr>
<td>WANG even</td>
<td></td>
<td>10.6</td>
<td>16.4</td>
</tr>
<tr>
<td>WANG odd</td>
<td></td>
<td>13.2</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Very similar databases

Training works well
Results for CalTech corpus

- classification task: object/no object
- 3 different databases: airplanes, faces, motorbikes, and backgrounds

<table>
<thead>
<tr>
<th>Method</th>
<th>error rates [%]</th>
<th>airplanes</th>
<th>faces</th>
<th>motorbikes</th>
</tr>
</thead>
<tbody>
<tr>
<td>32×32</td>
<td>24.0</td>
<td>15.0</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td>[WEBER00]</td>
<td>32.0</td>
<td>6.0</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>[FERGUS03]</td>
<td>9.8</td>
<td>3.6</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Tamura feature</td>
<td>1.6</td>
<td>3.9</td>
<td>7.4</td>
<td></td>
</tr>
<tr>
<td>$w_i = 1.0$</td>
<td>0.8</td>
<td>1.6</td>
<td>8.5</td>
<td></td>
</tr>
</tbody>
</table>
Results for MPEG-7

15 query images with known relevant results

Deselaers: Features for Image Retrieval

December 11th, 2003
Demonstration of
FIRE – Flexible Image Retrieval Engine
Improving Text-Based Image Retrieval Systems

Problem

Many image retrieval systems are text-based e.g. Google image search

Ambiguous words lead to unsatisfactory results. e.g. “cookie”

Idea

Use methods from computer vision and data mining to improve this situation

– Features & clustering

Results

Content-based clustering of results from Google image search

Content-based clustering of other databases to obtain quantitative results

– COIL
– WANG
Results from Google Image Search

(order: as from Google in March 2003)
Results for Clustering Google Images

Cluster 1

Cluster 2

Cluster 3

Cluster 4
## Results for Clustering COIL

<table>
<thead>
<tr>
<th>method/feature</th>
<th>Rand-index</th>
</tr>
</thead>
<tbody>
<tr>
<td>[KAESTER03] $k$-Means</td>
<td>0.53</td>
</tr>
<tr>
<td>[KAESTER03] hierarchical</td>
<td>0.62</td>
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<tr>
<td>(1) Invariant feature histogram</td>
<td>0.67</td>
</tr>
<tr>
<td>(2) Relational invariant feature histogram</td>
<td>0.63</td>
</tr>
<tr>
<td>(3) Tamura histogram</td>
<td>0.63</td>
</tr>
<tr>
<td>(1) &amp; (2) &amp; (3)</td>
<td><strong>0.82</strong></td>
</tr>
</tbody>
</table>
Conclusion

Development of a content-based image retrieval system
First quantitative results on different databases
First comparison of a wide variety of features
Comparison of different dissimilarity measures

Analysis of suitable feature sets:

Photographs, e.g. Corel
– Invariant feature histogram
– Tamura texture histogram
– Local features or local feature histograms
– Gabor features

Restricted domain, e.g. IRMA
– Pixel values
– Tamura texture features
– Local features or local feature histograms
– Gabor features
Conclusion

Analysis of performance measures
- Several performance measures available
- Performance measures strongly correlated
- ER as representative performance measure
- Standard database for comparison of CBIR systems missing

Connection between image retrieval and classification
- Classification methods can be used for image retrieval
- Image retrieval methods can be used for Classification
- Good results in one of the task lead to good results in the other

Improvement of text-based image search with clustering
- Connection between local features and image distortion model
Perspective

Starting point for comparison of image retrieval systems
Methods will be used in IRMA project
“Semantics-based” image retrieval: retrieve images based on objects or scenes depicted
Automatic annotation
Creation of a well-documented standard test for image retrieval
Features may be interesting for analysis of complex scenes
Database support
Thank You For Your Attention
Theory of Invariant Features

Given:

- Image $X$, $X(n_0, n_1) = \text{gray value at position } (n_0, n_1)$
- $g \in G$: transformation from a group of transformations
- $gX$: image $X$ transformed by $g$
- $f$: function from the image to a number: $f(X) : X \mapsto \mathbb{R}$

Then:

$$F(X) = \frac{1}{|G|} \int_{g \in G} f(gX)$$ is invariant against any transformation from $G$
Invariant Features in Practice

Properties

Let $G_{t,r}$ be the group of translations and rotations:

$g_{t_0,t_1,\varphi} \in G_{t,r}$ transforms the image by

$$(g_{t_0,t_1,\varphi}X)(n_0, n_1) = X(n'_0, n'_1)$$

with

$$\begin{pmatrix} n'_0 \\ n'_1 \end{pmatrix} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} n_0 \\ n_1 \end{pmatrix} + \begin{pmatrix} t_0 \\ t_1 \end{pmatrix}$$

The image $X(n_0, n_1)$ is discrete.

Choose e.g. $f(X) = \sqrt{X(1, 0)X(0, 2)}$

Results in

$$F(X) = \frac{1}{2\pi N_0 N_1} \sum_{t_0=1}^{N_0} \sum_{t_1=1}^{N_1} \sum_{r=1}^{R} \sqrt{X(\sin \frac{2\pi r}{R} + t_0, \cos \frac{2\pi r}{R} + t_1)X(2 \cos \frac{2\pi r}{R} + t_0, -2 \sin \frac{2\pi r}{R} + t_1)}$$

Extension to Scaling

Let $G_{t,r,s}$ be the group of translations, rotations, and scalings then

$$\begin{pmatrix} n'_0 \\ n'_1 \end{pmatrix} = s \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} n_0 \\ n_1 \end{pmatrix} + \begin{pmatrix} t_0 \\ t_1 \end{pmatrix}$$

$$F(X) = \frac{1}{2\pi N_0 N_1} \sum_{t_0=1}^{N_0} \sum_{t_1=1}^{N_1} \sum_{r=1}^{R} \sum_{s \in S} \sqrt{X(s \sin \frac{2\pi r}{R} + t_0, s \cos \frac{2\pi r}{R} + t_1)X(2s \cos \frac{2\pi r}{R} + t_0, -2s \sin \frac{2\pi r}{R} + t_1)}$$
Invariant Feature Histogram

Problem: Only one value per image (3 for RGB images) not rich enough

First solution to the problem of insufficient data
Replace one or more sums by histogramization:

\[ H_F(X) = \sum_{t_0=1,t_1=1}^{N_0,N_1} \text{hist}_{\text{hist}} \frac{1}{R} \sum_{r=1}^{R} f \left( g_{t_0,t_1,\frac{2\pi r}{R}} X \right) \]

RGB images
one 3-dimensional histogram or
3 one-dimensional histograms
Invariant Feature Vector

Second solution to the problem of insufficient data

Calculate several different invariant features instead of only one

Results in

\[ V = (v_1 \ldots v_N) \text{ a vector of invariant features} \]

with \( v_n = F_n(X) \)

and \( f_n \in \{ X(0, 0), \sqrt{X(0, 1)X(2, 0)}, \ldots \sqrt[3]{X(0, 0)X(1, 0)X(0, 4)} \} \)

RGB images: \( 3N \) values
Results for ZuBuD

- classification task: 201 buildings
- database of 1005 training and 115 test images

<table>
<thead>
<tr>
<th>Method</th>
<th>L1O ER[%] on training data</th>
<th>ER[%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[SHAO03]</td>
<td></td>
<td>13.9</td>
</tr>
<tr>
<td>[OBDRZALEK03]</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>(w_i = 1.0)</td>
<td>7.3</td>
<td>15.7</td>
</tr>
<tr>
<td>(w_i \in {0, \ldots, 10})</td>
<td>3.9</td>
<td>10.4</td>
</tr>
</tbody>
</table>
**rel-kernel function**

\[ f(X) = \text{rel}(X(n_0, n_1) - X(m_0, m_1)) \]

with

\[
\text{rel}(x) = \begin{cases} 
1 & \text{if } x < -c \\
\frac{1}{2c}(c - x) & \text{if } -c < x < c \\
0 & \text{if } x > c
\end{cases}
\]

is applied to intensity images

create a 3-bin fuzzy histogram counting lighter, equal, and darker

crate a 3D-histogram of 3-bin histograms

Resulting histogram is invariant to transformations and robust to brightness transformations.
Distance Measures

Minkowski Distances: \( D_p(H, H') = \left( \sum_{m=0}^{M-1} (H_m - H'_m)^p \right)^{\frac{1}{p}}. \)

Histogram Intersection: \( \cap(H, H') = \sum_{m=0}^{M-1} \min(H_m, H'_m) \)

Relative Deviation: \( D(H, H') = \frac{\sqrt{\sum_{m=0}^{M-1} (H_m - H'_m)^2}}{\frac{1}{2} (\sqrt{\sum_{m=0}^{M-1} H_m^2} + \sqrt{\sum_{m=0}^{M-1} H'_m^2})} \)

Relative Bin Deviation: \( D(H, H') = \sum_{m=0}^{M-1} \frac{\sqrt{(H_m - H'_m)^2}}{\frac{1}{2} (\sqrt{H_m^2} + \sqrt{H'_m^2})} \)

\( \chi^2 \)-Distance: \( \chi^2(H, H') = \sum_{m=0}^{M-1} \frac{H_m - H'_m}{H_m + H'_m} \)

Kullback Leibler Divergence: \( KL(H, H') = \sum_{m=0}^{M-1} H_m \log \frac{H_m}{H'_m} \)

Jensen Shannon Divergence: \( JD(H, H') = \sum_{m=0}^{M-1} H_m \log \frac{2H_m}{H_m + H'_m} + H'_m \log \frac{2H'_m}{H_m + H'_m} \)

Fidelity based distance measures: \( F(H, H') = \sum_{m=0}^{M-1} \sqrt{H_m} \sqrt{H'_m}, \quad F(H, H') = 1 - F(H, H'), \quad F_{\sqrt{-}}(H, H') = \sqrt{1 - F(H, H')}, \quad F_{\log}(H, H') = \log(2 - F(H, H')) \)

\( F_{\arccos}(H, H') = \frac{2}{\pi} \arccos F(H, H'), \quad F_{\sin}(H, H') = \sqrt{1 - F^2(H, H')} \)