Lehrstuhl für Informatik VI
Rheinisch–Westfälische Technische Hochschule Aachen
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1. Exercise Sheet **Pattern Recognition and Neural Networks**

The solutions to the problems indicated with (*...) may be submitted until **Wednesday, November 4th, 2009**, either in the secretariat of the *Lehrstuhl für Informatik VI* or at the latest before the exercise lesson on the same day.

To obtain the **Leistungsnachweis** (*Schein*) "Pattern Recognition and Neural Networks" the successful solution of 50% of the (*...)–problems is required.

The solutions to the problems can be submitted in groups of up to **three** students.

1. Many pattern recognition tasks deal with huge amounts of data, therefore numerical stability is an issue. A common task is the computation of variances. Several algorithms exist for this problem:

   2-pass algorithm:
   $$Var(X) = \frac{1}{N}\sum_{i=1}^{N}\Big(x_i - E(X)\Big)^2$$

   1-pass algorithm:
   $$Var(X) = \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)^2$$

   (a) Generate N Gaussian distributed random numbers with mean M and variance V. Apply the two algorithms for $N = 100, 100.000$, $M = 0.0, 100.000$ and $V = 1.0, 0.0001$ with single and double precision. The common measure for the error of floating point calculations is the relative error
   $$e_{\text{relative}} = \frac{|x - \tilde{x}|}{|x|},$$
   where x denotes the exact result and $\tilde{x}$ the result with rounding errors. Compute the relative errors. Which algorithm performs better? (* 4P)

   (b) How does the error depend on N, M and V? (* 4P)

2. **Expectation Properties**

   Simplify the following expressions for expectations using the definition of the expected value
   $$E\left\{f(x)\right\} = \int_{-\infty}^{\infty} f(x)\, p(x) dx$$
   with a probability density $p(x)$ and a function $f(x)$ of the random variable x.

   (a) $E\left\{(x-a)^2\right\} + 2a \cdot E\{x\}$ with some constant $a \in \mathbb{R}$.

   (b) $E\left\{\log \frac{x}{a}\right\}$ with some constant $a \in \mathbb{R}$. (* 2P)

   (c) $E\left\{E\{f(x)\}\right\}$. (* 2P)

Table 1: Joint probabilites for weather prediction

	high temperature			low temperature		
	sunny sky	cloudy sky	dark sky	sunny sky	cloudy sky	dark sky
rain	0.03	0.05	0.12	0.01	0.09	0.3
no rain	0.25	0.04	0.01	0.07	0.01	0.02

3. Weather Prediction

Almost everyone makes the expirience that it is possible to predict the weather on very simple observations for the near future. If you see a cloudy sky or it gets cold or it gets windy and so on, you can predict whether it will rain or not.

Table 1 contains the joint probabilities for the discrete variables rain, temperature and sky view.

Use the following abbreviations:
R+=rain R-=no rain
temperature: H= high L=low
sky: S=sunny C=cloudy D=dark

(a) Calculate the marginal probabilities for temperature.

(b) Calculate the marginal probabilities for rain and sky view. (* 2P)

(c) Calculate the conditional probabilities for rain given temperature and sky view. (* 4P)