Chair of Computer Science 6
RWTH Aachen University
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Automatic Speech Recognition

5. Exercise

Submission of the solutions: 02. 12. 2008 at the beginning of the lecture

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For the calculation, implementation and plotting of the following exercises you will need a mathematical toolkit. The public domain toolkit Octave is available at `http://www.octave.org`.

Task 5.1 Given the signal

$$s(t) = A\cos(\omega_0 t).$$

(a) Sketch the function $s(t)$ for $A = 1$ and $\omega_0 = 4$kHz on the interval $t \in [0, 0.01]$Hz. (1 P.)

(b) Determine the minimal sampling rate ω_S necessary for perfect reconstruction.

(c) Sketch the sampling procedure at sampling rates of $\omega_1 = \omega_S - 5$kHz and $\omega_2 = \omega_S + 5$kHz. (1 P.)

(d) Sketch the signals $s^{rec}_{\omega_1}(t)$ and $s^{rec}_{\omega_2}(t)$ reconstructed from samples sampled at ω_1 and ω_2. When applying the reconstruction formula, evaluate the infinite sum only for $n \in [-100, 100]$. Compare the reconstructed signals with the original one. (1 P.)

Task 5.2

The region of convergence is defined as the set of values of z so that the z-transform converges. The uniform convergence of the transform requires that the sequence is absolutely summable.

$$|X(z)| < \infty \text{ if } \quad \sum_{n=-\infty}^{+\infty} |x[n]||z|^{-n} < \infty$$

Determine the z-transform and its region of convergence for the following functions:

(a) $\left(\frac{1}{3}\right)^n u[n]$

(b) -$\left(\frac{1}{4}\right)^n u[-n-1]$

(c) $\left(\frac{1}{5}\right)^n u[-n]$

(d) $\delta[n]$

(e) $\delta[n-1]$

(f) $\delta[n+1]$

Task 5.3

A real time-continuous signal $s(t)$ is band-limited to $f_B = 5$kHz. The signal is sampled at a rate of 10000 samples per second. $X[k]$ values are calculated on the samples by DFT with a window length $N = 512$.

(a) Which frequencies do the following DFT coefficients belong to:
$X[0]$, $X[150]$, $X[256]$, $X[500]$? (1 P.)

(b) Can you state anything about the frequency $f = 4$kHz? (1 P.)

(c) Let us change the DFT length to $N = 2048$. Which frequency does $X[150]$ belong to? (1 P.)

$$\sum_{n=-\infty}^{\infty} |a_n|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |A(x)|^2 dx$$

Marc-Antoine Parseval des Chênes (1755-1836) was a French mathematician, most famous for what is now known as Parseval's theorem, which presaged the unitarity of the Fourier transform. His only mathematical publications were, apparently, five papers, published in 1806 as *Mémoires présentés à l'Institute des Sciences, Lettres et Arts, par divers savans, es lus dans ses assemblées*. It was in the second, 1799, memoir in which he stated, but did not prove (claiming it to be self-evident), the theorem that now bears his name. He further expanded upon it in his 1801 memoir, and used it to solve various differential equations.
Source: `http://www.wikipedia.com`