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Heuristics for planning with penalties and rewards formulated in logic and computed through circuits

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ABSTRACT

The automatic derivation of heuristic functions for guiding the search for plans is a fundamental technique in planning. The type of heuristics that have been considered so far, however, deal only with simple planning models where costs are associated with actions but not with states. In this work we address this limitation by formulating a more expressive planning model and a corresponding heuristic where preferences in the form of penalties and rewards are associated with fluents as well. The heuristic, that is a generalization of the well-known delete-relaxation heuristic, is admissible, informative, but intractable. Exploiting a correspondence between heuristics and preferred models, and a property of formulas compiled in d-DNNF, we show however that if a suitable relaxation of the domain, expressed as the strong completion of a logic program with *no time indices or horizon* is compiled into d-DNNF, the heuristic can be computed for *any* search state in time that is linear in the size of the compiled representation. This representation defines an evaluation network or circuit that maps states into heuristic values in linear-time. While this circuit may have exponential size in the worst case, as for OBDDs, this is not necessarily so. We report empirical results, discuss the application of the framework in settings where there are no goals but just preferences, and illustrate the versatility of the account by developing a new heuristic that overcomes limitations of delete-based relaxations through the use of valid but implicit plan constraints. In particular, for the Traveling Salesman Problem, the new heuristic captures the exact cost while the delete-relaxation heuristic, which is also exponential in the worst case, captures only the Minimum Spanning Tree lower bound.

1. Introduction

The automatic derivation of heuristic functions from problem descriptions in Strips and other action languages has been one of the key developments in recent planning research [14,51]. Provided with these heuristics, the search for plans becomes more focused, and if the heuristics are admissible (do not overestimate), the optimality of plans can be ensured [55]. The type of heuristics that have been considered so far, however, have serious limitations. Basically they are either non-admissible [12,40] or not sufficiently informative [39], and in either case they are restricted to cost functions where plan costs depend on actions but not on states. As a result, the tradeoffs that can be expressed are limited; in particular, it is not possible to state a preference for achieving or avoiding an atom p in the way to the goal, or take this preference into account when searching for plans.

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In this work, we address these limitations by formulating the derivation of heuristic functions in a logical framework. We have shown elsewhere that the heuristic represented by the planning graph [11] can be understood as a precise form of deductive inference over the stratified theory that encodes the problem [31]. Here our goal is not to reconstruct an existing heuristic but to use a logical formulation for producing a new one. The advantages of a logical framework are two: the derivation of heuristic information is an inference problem that can be made transparent with the tools of logic, and powerful algorithms have been developed that make certain types of logical inferences particularly effective. The latter includes algorithms for checking satisfiability [52], computing answer sets [61], and compiling CNF formulas into tractable representations [25].

Here we consider preferences over actions a and fluents p that are expressed in terms of real costs $c(a)$ and $c(p)$. Action costs are assumed to be non-negative, while fluent costs can be positive or negative. Negative costs express rewards. The cost of a plan is assumed to be given by the sum of the action costs plus the sum of the atom costs for the atoms made true by the plan. We are interested in computing a plan with minimum cost. This is a well defined task, which as we will see, remains well-defined even when *there are no goals* but just preferences. In such a case, the best plans simply try to collect rewards while avoiding penalties, and if there are no rewards, since action costs are non-negative, the best plan is empty.

The cost model is not fully general but is considerably more expressive than the one underlying classical planning. As we will see, the model generalizes recent formulations that deal with over-subscription or soft goals [60,64], which in our setting can be modeled as *terminal rewards*, rewards that are collected when the propositions hold at the end of the plan. On the other hand, the costs and rewards are combined additively, so unlike other recent frameworks [8], partially-ordered preferences are not handled.

The definition of the planning model is motivated by the desire to have additional expressive power and a principled and feasible computational approach for dealing with it. For this, we want a useful heuristic, with a clear semantics, capable of capturing interesting cost tradeoffs, and a feasible algorithm for computing it. We will be able to express in the model, for example, navigation problems where coins of different values are to be collected by avoiding as much as possible certain cells, or blocks-world problems where a tallest tower is to be constructed, or where the number of blocks that touch the table is to be minimized. In order to test the effectiveness of the approach we will also consider classical planning tasks where we will assess the approach empirically in relation to existing heuristics and planners.

The heuristic h_c^+ that we develop is simple and corresponds to the optimal cost of the relaxed problem where the delete-lists of all actions are ignored [12]. Since searching with this heuristic, even in the classical setting, involves *an intractable computation in every state s visited* [15], planners such as HSP and FF resort to polynomial but non-admissible approximations [12,40]. In this work, while considering the more general cost structure, we take a different approach: we compute the heuristic h_c^+ for each search state, but pay the price of *an intractable computation only once*, as preprocessing. This preprocessing yields what can be deemed as an *evaluation network* or *circuit* where we can plug *any* search state and obtain its heuristic value in linear time. Of course, the time to construct this evaluation network and the size of the network may both be exponential, yet this is not necessarily so. The evaluation network, indeed, is nothing else but the directed acyclic graph that results from compiling a relaxation of the planning theory into d-DNNF, a form akin to OBDDs introduced in [21,22] that renders efficient a number of otherwise intractable queries and transformations [25]. The heuristic values are then obtained as the cost of the 'best' models, which can be computed in linear time once the relaxed theory is compiled into d-DNNF [26].

The framework defined by the formulation of the heuristic h_c^+ in terms of logic and their computation in terms of compiled d-DNNF representations is then evaluated empirically over a broad set of problems, where the heuristic is used to guide the search for optimal plans.

An important characteristic of the logical encoding of the delete-relaxation heuristic is that unlike the standard logical encodings of planning problem [44], *no explicit temporal stratification* in the form of time indices or horizons is needed. This follows from the use of *positive logic programs* for expressing the effects of the actions as an intermediate representation, and the focus on the models that are *minimal* in the sense that true fluents must have a well-founded justification. Such minimal models capture an *implicit stratification* that is in correspondence with the explicit temporal stratification adopted by the standard logical approaches to planning. A concrete result of this implicit stratification is that the resulting heuristic estimates the true optimal cost of the problem, and not the optimal cost given a specific temporal horizon.

The paper is a revised version of [13] where the results are extended to a new class of heuristics that complement the use of the delete-relaxation with valid *plan constraints*.[1] Valid plan constraints are formulas defined over the set of action and fluent symbols that are satisfied by some optimal plan. Making valid plan constraints explicit in a problem hence does not affect the true cost of a problem but can boost the value of the heuristic while keeping it admissible. We show for example the new heuristic is optimal for problems like the Traveling Salesman Problem (TSP), where the delete-relaxation heuristic h_c^+, which is also exponential in the worst case (unless P = NP), yields only the Minimum Spanning Tree (MST) lower bound (for references on the TSP and MST; see [18,48,56]).

The plan for the paper is the following: we present in order the cost model, the heuristic h_c^+, the correspondence between h_c^+ and the rank of a suitable propositional theory, and the computation of the heuristic for any state in terms

[1] We also correct a mistake in [13] where a planning language with conditional effects is used although some of the results apply only to Strips.

of a suitable d-DNNF compilation of the theory. We then deal with the search algorithm, that must handle negative costs, present the experimental results, and consider the more powerful heuristic that arises when plan constraints are taken into account. We then summarize the main contributions, and discuss related work and open problems.

2. Planning and cost model

We consider Strips planning problems $P = \langle F, I, O, G\rangle$ where F is the set of relevant atoms or fluents, $I \subseteq F$ and $G \subseteq F$ are the initial and goal situations, and O is a set of (grounded) actions a with precondition, add, and delete lists $Pre(a)$, $Add(a)$, and $Del(a)$.

A plan π for a problem $P = \langle F, I, O, G\rangle$ is an applicable action sequence $a_0, a_1, \ldots, a_n$ with $a_i \in O$ for $i = 0, \ldots, n$, that transforms the initial state s_0 associated with I into a final state s_{n+1} where the goal G holds. States are sets of fluents, the initial state s_0 is I, and the state s_{i+1} that follows action a_i in state s_i is $s_{i+1} = s_i + Add(a_i) - Del(a_i)$, where '+' and '−' stand for set union and difference respectively. The action a_i is applicable in s_i if $Pre(a_i) \subseteq s_i$, and the action sequence $a_0, a_1, \ldots, a_n$ is applicable if each action is applicable in the state that results from the previous actions in the sequence.

We are interested in plans π for P that minimize a cost measure $c(\pi)$. The cost $c(\pi)$ of a plan π in classical planning is associated with the *number of actions* in the plan; a cost measure that is usually written as $|\pi|$ and can also be expressed as:

$$c(\pi) = \sum_{a_i \in \pi} c \tag{1}$$

where c is a positive constant equal to 1. This cost structure however is often too limited. An immediate generalization can be obtained by assuming that actions a have a non-uniform and non-negative cost $c(a)$ so that the cost of a plan becomes:

$$c(\pi) = \sum_{a_i \in \pi} c(a_i). \tag{2}$$

The classical cost structure follows then by setting the action costs $c(a)$ to 1. Interestingly, some of the heuristics developed for classical planning, including the additive [12] and h^m heuristics [39], deal easily with non-uniform action costs, while others, such as the heuristics underlying Graphplan [11] and the FF planner [40], which are defined in terms of planning graphs, do not.

A further generalization can be obtained by making costs dependent not only on the actions made in the plan, but also on the *states* that are traversed:

$$c(\pi) = \sum_{a_i \in \pi} c(a_i, s_i). \tag{3}$$

Here $c(a_i, s_i)$ stands for the cost of executing action a_i in the state s_i that results from the execution of the previous actions in the plan.

In the general cost model captured by (3), costs may depend on both actions and states, in (2), they may depend on the actions only, while in (1) they may depend on neither one. In this work, we deal with plan costs that depend on both the actions and the states but in the *restricted* form:

$$c(\pi) = \sum_{a_i \in \pi} c(a_i) + \sum_{p \in F(\pi)} c(p) \tag{4}$$

where $F(\pi)$ is the set of fluents made true by plan π *at any time point during the plan execution*, and $c(p)$ is the cost of fluent $p \in F$.

In comparison with (3), the cost model given by (4) defines the costs $c(a_i, s_i)$ *additively* in terms of the action costs $c(a_i)$ and fluent costs $c(p)$. The costs $c(a)$ of actions is assumed to be non-negative, while the costs $c(p)$ of fluents can be *positive*, *negative*, or *zero* (by default). Positive fluents costs are called *penalties*, while negative fluents costs are called *rewards*.

For optimal plans to have always a bounded cost, the plan cost measure $c(\pi)$ defined by (4) counts fluent costs $c(p)$ at most once.[2] Without this restriction, a plan could get an infinite reward by achieving an atom p with negative cost $c(p)$ and then 'waiting' doing something irrelevant.

Given the cost of plans $c(\pi)$ captured by (4), we are interested in the plans π that minimize $c(\pi)$; these are the optimal or best plans. If there is a plan at all, this optimization problem is well defined, although the best plan is not necessarily unique. We denote by $c^*(P)$ the cost of a best plan for problem P with respect to the cost function c over actions and fluents

$$c^*(P) \stackrel{\text{def}}{=} \min\{c(\pi):\ \pi \text{ is a plan for } P\} \tag{5}$$

[2] This restriction makes the model *non-markovian* in the sense that the contribution of action a_i in the state s_i is $c(a_i)$ plus the cost $c(p)$ of the atoms p in the next state s_{i+1} that have not been true in earlier states. We will say more about this when discussing the search for optimal solutions in this model and the information that must be kept in the search nodes.

and set $c^*(P)$ to ∞ when P admits no plan. Clearly, when $c(a) = 1$ and $c(p) = 0$ for all actions a and fluents p, the cost criterion of classical planning is obtained where $c^*(P)$ measures the minimum number of actions needed to solve P. The resulting framework, however, is more general, as costs on both actions and fluents can be expressed, and the latter can be either positive or negative. Indeed, it is possible to model problems with no goals but just preferences expressed in the form of rewards. In such a case, the empty plan is optimal if there are no rewards (action costs are assumed to be non-negative), but other plans may have a smaller, and thus negative cost, when rewards are present. In general, the best plans must achieve the goal by trading off action and fluent costs.

We will call the planning cost model captured by (4) the *penalties and reward* model, abbreviated as PR. This cost model is similar to the one used in over-subscription planning where due to constraints or preferences, it may not be possible or convenient to achieve all the goals [60,64]. There are two important differences though. The first is that in PR atoms can be rewarded when they are achieved *anytime* during the execution of the plan, not only when they are achieved at the *end* of the plan. The second is that such atoms may express either penalties or rewards. If they express penalties (positive costs), they are not atoms to be achieved but to be avoided during the execution.

In order to capture preferences on *end states* as opposed to preferences on intermediate states, when required, we consider the use of an special *End* action with zero cost that must terminate all plans, whose preconditions are the goals G of the problem, and whose effect is a dummy goal *done*. In order for such an action to terminate all plans it is sufficient to have an additional fluent *not_done*, initially true, that is a precondition of all actions, and that is deleted by the action *End*. With this convention, the representation of preferences on end states becomes possible by simply adding *conditional effects* to the action *End*. While we assume that the language is Strips and hence that conditional effects are not accommodated, the generalization to such an extension (a final action with conditional effects) is straightforward and is supported in the planner.

3. Modeling

The cost model for planning formulated above is simple but flexible. Some preference patterns that can easily be expressed are:

- **Terminal Costs:** an atom p can be rewarded or penalized if true *at the end* of the plan by introducing a new atom p', initialized to false along with the conditional effect $p \rightarrow p'$ for the action *End*. A reward or penalty $c(p')$ on p' then captures a reward or penalty on p at the end of the plan. In such a case, we call $c(p')$ a *terminal cost* on p and p' a *terminal atom*.
- **Goals:** once costs on terminal states can be expressed, goals are not strictly required. Semantically, a hard goal can be modeled as a sufficiently high terminal reward. Computationally, however, the first option will usually yield better results.
- **Soft Goals:** soft goals can be modeled as terminal rewards, and the best plans will achieve them depending on the costs involved.
- **Preferences on Literals:** while the model assumes that costs are associated with positive literals p but no negative ones, standard planning transformation techniques can be used to add a new atom p' that is true exactly when p is false [33,53]. Preferences on the negation of p can then be expressed as preferences on p'.
- **Rewards on Conjunctions:** it is possible to reward states in which a set of atoms $p_1, \ldots, p_n$ is true by means of an action $Collect(p_1, \ldots, p_n)$ with preconditions $p_1, \ldots, p_n$, and effect p, where p is a new atom that is rewarded. The same trick however does not work for expressing *penalties on conjunctions*. The reason is that optimal plans will choose to collect a free reward if possible, but will never choose to collect a free cost (as would be required if the atom p were a penalty and not a reward).

As an illustration, a blocks-world problem where the number of blocks that touch the table is to be kept to a minimum (at the price of obtaining possibly a longer plan) can be obtained by penalizing the atoms $on(x, table)$ for blocks x. More interestingly, the problem of building the tallest possible tower results from assigning *terminal* rewards to the atoms $on(x, y)$ for all the blocks x and y (with *non-terminal* rewards instead, the best plans would move the blocks around placing every block on top of every other block to collect all rewards associated with the atoms $on(x, y)$). If actions have positive costs, the best plans will be the ones that achieve a tallest tower in a minimum number of steps (i.e., choosing one of the existing tallest towers as the basis). Likewise, problems where an agent is supposed to pick up some coins while avoiding a dangerous 'wumpus', can be modeled by rewarding the atoms $have(coin_i)$ and penalizing the atoms $at(x, y)$ where x, y is the position of the wumpus.[3]

Among preference patterns that the PR cost model does *not* capture in a natural way are positive costs on sets of atoms (mentioned above) and partial preferences where certain costs are not comparable [6,8].

The PR model can be extended to deal with repeated penalties or rewards, as when a cost is paid each time an atom is made true. We do not consider such an extension in this work, however, for two reasons: semantically, with repeated

[3] The Wumpus problem in [59] is more interesting though as it involves uncertainty and partial observability, issues that are not addressed in the PR model.

rewards, some problems do not have a well-defined cost (cyclic plans may accumulate an infinite reward);[4] and computationally, the proposed heuristics do not capture the specific features of such a model, even if as we will see, they would remain admissible then.

4. Heuristic h_c^+

Heuristics are fundamental for searching in large spaces. In the classical setting, several effective heuristics have been proposed, most of which are defined in terms of the delete-relaxation: a simplification of the problem where the delete-lists of the operators are dropped. Delete-free planning is simpler than planning in the sense that plans can be generated in polynomial time; still *optimal* delete-free planning is intractable too [15]. Thus, on top of this relaxation, the heuristics used in many classical planners rely on other simplifications; the formulation in [40] drops the optimality requirement in the relaxed problem, while the one in [14,51], assumes that subgoals are independent. In both cases, the resulting heuristics are not admissible.

The heuristic that we formulate for the PR model builds on and extends the optimal delete-relaxation heuristic proposed in classical planning. If P^+ is the delete-relaxation of problem P, i.e., the planning problem obtained by dropping all the delete-lists from the actions in P, the heuristic $h_c^+(P)$ that provides an estimate of the cost of solving P given the cost function c is defined as

$$h_c^+(P) \stackrel{\text{def}}{=} c^*(P^+), \tag{6}$$

where $c^*(P^+)$ is the optimal cost of the delete-relaxation. For the 0/1 cost function that characterizes classical planning, where the cost of all atoms is 0 and the cost of all actions is 1, this definition yields the (optimal) delete-relaxation heuristic which provides an estimate on the number of steps to the goal. This heuristic is admissible and tends to be quite informative too (see the empirical analysis in [41]). Expression (6) generalizes this heuristic to the larger class of cost functions where actions may have non-uniform costs and atoms can be rewarded or penalized, and where it remains admissible too:[5]

Proposition 1 *(Admissibility). The heuristic $h_c^+(P)$ is* admissible; *i.e. $h_c^+(P) \leqslant c^*(P)$.*

If we let $P[I = s]$ and $P[G = g]$ refer to the planning problems that are like P but with initial and goal situations $I = s$ and $G = g$ respectively, then (optimal) *forward* heuristic-search planners aimed at solving P need to compute the heuristic values $h_c^+(P[I = s])$ for *all* states s encountered, while *regression* planners need to compute the heuristic values $h_c^+(P[G = g])$ for *all* encountered subgoals g. Since each such computation is intractable, even for the 0/1 cost function, classical planners like HSP and FF settle on polynomial but non-admissible approximations. In this work we take a different path: we use the h_c^+ heuristic in the more general cost setting, but rather than performing an intractable computation for *every search state* encountered, we perform an intractable computation *only once*. For this, we establish a correspondence between heuristic values and ranks of a propositional theory, which can be computed in polynomial time provided that the theory is compiled in a suitable form.

5. Heuristics, preferred models, and d-DNNF

Following [43,44], a propositional encoding of a sequential planning problem $P = \langle F, I, O, G\rangle$ with horizon n can be obtained by introducing fluent and action variables p_i and a_i for each fluent p, action a, and time step i in a theory $T_n(P)$ comprised of the following formulas:

1. **Init:** p_0 for $p \in I$, $\neg q_0$ for $q \in F - I$
2. **Goal:** p_n for $p \in G$
3. **Actions:** For $i = 0, 1, \ldots, n-1$ and all $a \in O$
   $a_i \supset p_i$ for $p \in Pre(a)$
   $a_i \supset q_{i+1}$ for each positive effect $q \in Add(a)$
   $a_i \supset \neg q_{i+1}$ for each negative effect $q \in Del(a)$
4. **Frame:** For $i = 0, \ldots, n-1$ and all $p \in F$
   $p_i \wedge \big(\bigwedge_{a:p\in Del(a)} \neg a_i\big) \supset p_{i+1}$
   $\neg p_i \wedge \big(\bigwedge_{a:p\in Add(a)} \neg a_i\big) \supset \neg p_{i+1}$
5. **Seriality:** For $i = 0, 1, \ldots, n-1$ and $a \neq a'$, $\neg(a_i \wedge a'_i)$.

For a sufficiently large horizon n, the models of the propositional theory $T_n(P)$ are in correspondence with the plans for P: each model encodes a plan, and each plan determines a model.

[4] This same problem arises in Markov Decision Processes where the usual work around is to discount future costs [10].

[5] Formal proofs of the results in the paper can be found in Appendix A.

For any cost function c, if we define the *rank* $r(M)$ *of a model* M as the cost $c(\pi)$ of the plan that the model M makes true, and define the *rank* $r^*(T)$ *of a theory* T as the rank of its best model

$$r^*(T) \stackrel{\text{def}}{=} \min_{M \models T} r(M) \tag{7}$$

with $r^*(T) = \infty$ if T has no models, it follows then that the *cost* of P and the *rank* of its propositional encoding $T_n(P)$ are related as follows:

Proposition 2 *(Costs and Ranks). For a sufficiently large time horizon* n *(exponential in the worst case),* $c^*(P) = r^*(T_n(P))$*, where the rank* $r(M)$ *of a model* M *of* $T_n(P)$ *is given by the cost of the plan defined by* M*.*

This correspondence between the cost of a planning problem and the rank of a propositional theory, follows directly from the definitions and does not give us much unless we have a way to derive *theory ranks* effectively. A result in this direction comes from [26] that shows how to compute theory ranks $r^*(T)$ efficiently when r is a *literal-ranking* function and the theory T is in d-DNNF [22]. A literal ranking function ranks models in terms of the rank of the *literals* l that they render true:[6]

$$r(M) \stackrel{\text{def}}{=} \sum_{l:M \models l} r(l). \tag{8}$$

For literal-ranking functions r and propositional theories T compiled into d-DNNF, Darwiche and Marquis show that

Theorem 3 *(Darwiche and Marquis). If a propositional theory* T *is in d-DNNF and* r *is a literal-ranking function, then the rank* $r^*(T)$ *can be computed in time linear in the size of* T*.*

This result suggests that we could compute the optimal cost $c^*(P)$ of P by compiling first the theory $T_n(P)$ into d-DNNF and then computing its rank $r^*(T_n(P))$ in time linear in the size of the compilation. There are two obstacles to this however. The first is that the model ranking function $r(M) = c(\pi(M))$ in Theorem 2 is defined in terms of the cost of the atoms made true during the execution of the plan, not in terms of the literals true in the model, and hence it is not exactly a *literal*-ranking function such as (8). The second, and more critical, is that the horizon n needed for ensuring Theorem 2 is normally too large for $T_n(P)$ to compile. We show below though that these two problems can be handled better when the computation of the heuristic $h^+_c(P)$, that approximates the real cost $c^*(P)$, is considered instead.

Before focusing on the logical encodings required for computing the heuristics $h^+_c(P[I = s, G = g])$ for any state s and subgoal g in linear-time from a suitable d-DNNF compilation, let us briefly recall how d-DNNF formulas T are represented and how their ranks $r^*(T)$, defined by (7) and (8), are computed. A formula T in d-DNNF is a rooted DAG (Directed Acyclic Graph) whose leaves are the positive and negative literals associated with the variables in T along with the constants **true** and **false**, and whose internal nodes stand for conjunctions or disjunctions (AND and OR nodes, respectively). The formula $wff(n)$ associated with the root node n of a d-DNNF formula can be read off recursively from the leaves as follows:

$$wff(n) = \begin{cases} L & \text{if } n \text{ is a leaf associated with literal } L \\ \bigwedge_i wff(n_i) & \text{if } n \text{ is an AND node with children } n_i \\ \bigvee_i wff(n_i) & \text{if } n \text{ is an OR node with children } n_i. \end{cases} \tag{9}$$

A d-DNNF formula is thus in Negated Normal Form (NNF) as it contains only the connectives for conjunctions, disjunctions, and negations, and negation occurs only in literals [4]. The d-DNNF formula is a NNF formula represented as a DAG that satisfies *two conditions.* The first is *decomposability*, that accounts for the 'D' in d-DNNF and requires that the subformulas associated with the children of an AND node, share no variables. The second is *determinism*, that accounts for the 'd' in d-DNNF and requires that the subformulas associated with the children of an OR node, be mutually exclusive.

These two conditions enable a large number of otherwise intractable queries and transformations to be done in time which is linear in the size of the DAG representation [25]. For example, the procedure for computing the rank $r^*(T)$ of a formula T in d-DNNF can be expressed in terms of the value of the function $r^*(n)$ computed recursively, bottom up as follows [26]:

$$r^*(n) = \begin{cases} 0 & \text{if } n = \textbf{true} \\ \infty & \text{if } n = \textbf{false} \\ r(L) & \text{if } n = L \text{ where } L \text{ is a literal different than } \textbf{true} \text{ and } \textbf{false} \\ \sum_i r^*(n_i) & \text{if } n \text{ is an AND node with children } n_i \\ \min_i r^*(n_i) & \text{if } n \text{ is an OR node with children } n_i. \end{cases} \tag{10}$$

[6] Darwiche and Marquis use the name 'normal weighted bases' rather than literal-ranking functions.

The DAG representing the theory T in d-DNNF thus becomes an *arithmetic circuit* with the leaves replaced by the numbers 0, ∞, or $r(L)$ according to whether the leaf is the constant **true**, **false**, or the non-constant literal L, and the internal nodes replaced by sums and minimizations, according to whether they stand for AND or OR nodes. The output of this circuit, computed in time that is linear in the size of the circuit, is the theory rank $r^*(T)$. For computing in turn the rank $r^*(T \cup S)$ of the theory that extends T with a set S of literals over the same language, the same bottom-up procedure working on the compiled representation of T can be used, treating the leaves $n = L$ as **true** if $L \in S$, and as **false** if $\neg L \in S$ [26].

Any formula can be compiled into d-DNNF [23]. The time to build the compiled representation and the size of the compiled representation may both be exponential in the worst case, yet like compilation into OBDDs, a compiled logical representation commonly used in automated verification [16] and closely related to d-DNNF's [25], this is not necessarily so in general. Indeed, the compilation of the theory is exponential (in both time and space in the worst case) in a structural parameter associated with the theory known as the treewidth, which measures the degree of interaction among its variables; see [21,24].

Once a correspondence between the heuristic $h_c^+(P)$ and the rank $r^*(T)$ of a suitable propositional encoding is established, we will see that an arithmetic circuit like that one described above can be used to map *any* state s and subgoal g into the heuristic value $h_c^+(P[I = s, G = g])$.

5.1. Stratified encodings

Since the heuristic $h_c^+(P)$ is defined in terms of the optimal cost of the relaxed, delete-free problem P^+, it is natural to consider the computation of the heuristic in terms of the theory $T_n(P^+)$ of the relaxed problem. We will do this below but first we will simplify the theory $T_n(P^+)$ by *dropping the seriality constraints* that are no longer needed in the delete-free setting where any parallel Strips plan can be serialized retaining its cost. In addition, we will drop from $T_n(P^+)$ the *init* and *goal clauses* as we want to be able to compute the heuristic values $h_c^+(P[I = s])$ and $h_c^+(P[G = g])$ for *any* possible initial state s and subgoals g that might arise in a progression or regression search respectively. We call the set of clauses that are left in $T_n(P^+)$, the *stratified (relaxed) encoding* and denote it by $T_n^+(P)$. Later on we will consider another encoding that does not involve a temporal stratification at all.

The first crucial difference between the problem P and its delete-free relaxation P^+ is the horizon needed for having a correspondence between models and plans. For P, the optimal plans may have exponential length due to the number of different states that a plan may visit. On the other hand, the optimal plans for P^+ have at most *linear* length, as without deletes, actions can only add atoms, and thus the number of different states that can be visited is bounded linearly by the number of fluents.

The second difference is that the optimal cost of the delete-free problem can be put in correspondence with the rank of its propositional encoding using a simple *literal-ranking* function compatible with Theorem 3, as any atom achieved in a delete-free plan remains true until the end of the plan.

If we let I_0 and G_n stand for the init and goal clauses in the theory $T_n(P)$ that capture the initial and goal situations $I = s$ and $G = g$ respectively, the following correspondence between heuristic values and theory ranks can be established:

Proposition 4 *(Heuristics and Ranks). For a sufficiently large horizon n (linear in the worst case) and any initial and goal situations s and g,*

$$h_c^+\big(P[I = s, G = g]\big) = r^*\big(T_n^+(P) \cup I_0 \cup G_n\big),$$

where r is the literal ranking function such that $r(p_n) = c(p)$ for every fluent p, $r(a_i) = c(a)$ for every action a and $i \in [0, n-1]$, otherwise $r(l) = 0$.

Exploiting then Theorem 3 and the ability of d-DNNF formulas to be conjoined with *literals* in linear-time, we get:

Theorem 5 *(Compilation and Heuristics). Let $\Pi_n(P)$ refer to the compilation of theory $T_n^+(P)$ into d-DNNF where n is a sufficiently large horizon (linear in the worst case). Then the heuristic values $h_c^+(P[I = s, G = g])$ for any initial and goal situations s and g, and any* cost function *c, can be computed from $\Pi_n(P)$ in linear time.*

This theorem tells us that a single compilation suffices for computing a huge set of heuristic values in time that is linear in the size of the compilation. The heuristic values $h_c^+(P[I = s, G = g])$ provide estimates of the cost of achieving any goal g from any initial state s. During a forward search, however, only the values $h_c^+(P[I = s])$ are needed, while in a regression search, only the values $h_c^+(P[G = g])$ are needed. The formulation, however, yields a larger number of heuristic values that can be used, for example, in a bidirectional search.

5.2. Logic programming encodings

The encoding $T_n(P)$ for computing the optimal cost $c^*(P)$ of P requires an horizon n that is exponential in the worst case, while the encoding $T_n^+(P)$ for computing the heuristic $h_c^+(P)$ requires an horizon that is only linear. Still, a more

compact encoding for computing h_c^+, which requires *no time or horizon at all*, can be obtained. We call it the LP encoding as it is obtained from a set of positive Horn clauses [47].

The LP encoding of a planning problem P for computing the heuristic h_c^+ is obtained from the propositional LP rules of the form

$$p \leftarrow Pre(a), a \tag{11}$$

for each positive effect $p \in Add(a)$ associated with an action a with preconditions $Pre(a)$ in P. For convenience, as we explain below, for each atom p in P, we introduce also a 'dummy' action $set(p)$ with unique effect p and no precondition encoded as:

$$p \leftarrow set(p). \tag{12}$$

These actions will be formal devices for 'setting' the initial situation to s when computing the heuristic values $h_c^+(P[I = s])$ in a progression search. No such encoding trick is needed for the goals g in a regression search.

The LP encoding, that will enable us to compute the h_c^+ heuristic in a more effective way, has two features that distinguish it from the previous stratified encodings. The first is that there are no time indices. These indices are not necessary as we will focus on a class of *minimal* models of the program that have an *implicit stratification* which is in correspondence with the *temporal stratification*. Such minimal models will be grounded on the actions as all fluents will be required to have a well-founded support based on them. The second distinctive feature is that actions do not imply their preconditions. This will not be a problem either as actions all have non-negative costs and, in this encoding, all require their preconditions in order to have some effect. So while models that make actions true without their preconditions are possible, such models will not be preferred over the same models where such actions are false, and hence they will not affect the rank of the theory.

For a planning problem P, let $L(P)$ refer to the collection of rules (11) and (12) encoding the effects of the actions in P, including the $set(p)$ actions, and let $wffc(L(P))$ stand for the *well-founded fluent completion* of $L(P)$: a completion formula defined below that forces each fluent p to have a well-founded support. Then if we let $I(s)$ refer to the collection of unit clauses over the variables $set(p)$ that represent a situation s, namely $set(p) \in I(s)$ iff $p \in s$, and $\neg set(p) \in I(s)$ iff $p \notin s$, we obtain that the correspondence between heuristic values and LP encodings becomes:

Proposition 6 *(Heuristics and Ranks). For any initial situation s, goal g, and cost functions c,*

$$h_c^+\big(P[I = s, G = g]\big) = r^*\big(wffc\big(L(P)\big) \cup I(s) \cup g\big)$$

where r is the literal ranking function such that $r(l) = c(l)$ for positive literals l and $r(l) = 0$ otherwise.

From this result and the properties of d-DNNF formula, we obtain:

Theorem 7 *(Main). Let $\Pi(P)$ refer to the compilation of theory $wffc(L(P))$ into d-DNNF. Then* for any initial and goal situations s and g, *and any* cost function c, *the heuristic value $h_c^+(P[I = s, G = g])$ can be computed from $\Pi(P)$ in linear time.*

The well-founded fluent completion $wffc(L(P))$ picks up the models of the logic program $L(P)$ that are *minimal in the set of fluents given the actions in the model*; namely the minimal models of the logic program $L(P) \cup A$ for any set of actions A with the actions in A treated as facts. In such models, fluents have a non-circular support that is based on the actions that are true in the model. In particular, if $L(P)$ is an *acyclic* program, $wffc(L(P))$ is nothing else but Clark's completion applied to the fluents [1,17]. The program $L(P)$ is acyclic if the directed graph formed by connecting every atom that appears in the body of a rule to the atom that appears in the head, is acyclic; and Clark's completion applied to the fluent literals adds the formulas

$$p \supset B_1 \vee \cdots \vee B_n$$

to each fluent p with rules

$$p \leftarrow B_i$$

for $i = 1, \ldots, n$ in $L(P)$, and the formula $\neg p$ when there are no rules for p at all.

In the presence of cycles in $L(P)$, the well-founded fluent completion $wffc(L(P))$ does not reduce to Clark's completion, which does not exclude circular supports. In order to rule out circular supports and ensure that the fluents in the model can be stratified as in temporal encodings, a stronger completion is needed. Fortunately, this problem has been addressed in the literature on Answer Set Programming [2,5,34] where techniques have been developed for translating cyclic and acyclic logic programs into propositional theories whose models are in correspondence with their Answer Sets [9,49]. The logic program $L(P)$ is a positive logic program whose unique minimal model, for any set of actions, coincides with its unique Answer Set. The strong completion $wffc(L(P))$ can thus be obtained from any such translation scheme, with the provision that only fluent atoms are completed (not actions). In our current implementation, we follow the scheme presented in [50] that introduces new atoms and new rules that provide a consistent, partial ordering on the fluents in $L(P)$ so that the resulting models become those in which the fluents have well-founded, non-circular justifications. This is a polynomial transformation which is illustrated in the example below. From now on, $wffc(L(P))$ will refer to the result of such a translation.

6. Example

As an illustration of the logical formulation of the delete-relaxation heuristic h_c^+, consider a simple problem P that involves three locations $A \leftrightarrow B \leftrightarrow C$, such that an agent can move between A and B, between B and C, but cannot move directly between A and C.

This problem can be modeled with actions of the form $move(x, y)$ with precondition $at(x)$ and effects $at(y)$ and $\neg at(x)$, for x and y ranging over A, B, and C so that the resulting actions correspond to the allowed transitions. For each such action in P, $L(P)$ contains rules

$$at(y) \leftarrow at(x), move(x, y)$$

along with

$$at(y) \leftarrow set\big(at(y)\big).$$

Consider now an initial state $s = \{at(A)\}$, a goal $g = \{at(C)\}$, and a cost function $c(a) = 1$ for all actions except for $move(A, B)$ with cost $c(move(A, B)) = 10$.

The best plan for this state-goal pair in the delete-relaxation is $\pi = \{move(A, B), move(B, C)\}$, which is also the best plan without the relaxation, so

$$h_c^+\big(P[I = s, G = g]\big) = c^*\big(P[I = s, G = g]\big) = 11.$$

Proposition 6 says that this heuristic value must correspond to the rank of the well-founded fluent completion of $L(P)$, $wffc(L(P))$, extended with the set of literals given by

$$I(s) = \big\{set\big(at(A)\big), \neg set\big(at(B)\big), \neg set\big(at(C)\big)\big\},$$

and

$$g = \big\{at(C)\big\}.$$

In order to get an intuition for this completion, let us illustrate first why it must be stronger than Clark's completion. For this problem, Clark's completion for the fluent atoms gives us the theory:

$$at(C) \equiv \big(at(B) \wedge move(B, C)\big) \vee set\big(at(C)\big)$$
$$at(B) \equiv \big(at(A) \wedge move(A, B)\big) \vee set\big(at(B)\big) \vee \big(at(C) \wedge move(C, B)\big)$$
$$at(A) \equiv \big(at(B) \wedge move(B, A)\big) \vee set\big(at(A)\big).$$

For the literal ranking function r that corresponds to c,[7] the best ranked model of Clark's completion extended with the literals in $I(s)$ and g, has rank 2 which is different than $h_c^+(P[I = s, G = g]) = 11$. In such a model, the costly $move(A, B)$ action is avoided, and the fluent $at(C)$ has a circular justification that involves the cheaper actions $move(B, C)$ and $move(C, B)$. This arises because the program $L(P)$ contains a cycle involving the atoms $at(B)$ and $at(C)$.

In the well-founded completion defined in [50], Clark's completion is applied to a program which is different than $L(P)$ and where circularities are broken. For this example, each rule instance

$$at(y) \leftarrow at(x), move(x, y)$$

in $L(P)$ is replaced by a collection of rules

$$r_k \leftarrow NOT\, at(y) \prec at(x), at(x), move(x, y)$$
$$at(y) \leftarrow r_k$$
$$at(x) \prec at(y) \leftarrow r_k$$
$$at(z) \prec at(y) \leftarrow r_k, at(z) \prec at(x)$$

where z ranges over the locations A, B, and C, and 'NOT' stands for negation as failure. The new rules introduce two new predicates: one is r_k that stands for a unique identifier of the original rule instance in $L(P)$; the other is '$\prec$' that represents a precedence constraint among the atoms $at(A)$, $at(B)$, and $at(C)$ that form a loop in $L(P)$ and ensures that the supports in the resulting models are all well-founded. Thus, the first pair of rules allows the atoms $at(x)$ and $move(x, y)$ to support the atom $at(y)$ when $at(x)$ does not precede $at(y)$, while the third rule ensures that this support makes $at(x)$ precede $at(y)$, and the last rule that the precedence relation is closed under transitivity.

The well-founded fluent completion $wffc(L(P))$ is the result of applying Clark's completion to the fluents of the resulting transformed program. The best model of such a theory extended with the set of literals in $I(s)$ and g as above, contains

[7] From Proposition 6, $r(l) = c(l)$ if l is a positive literal and $r(l) = 0$ otherwise.

the actions $move(A, B)$ and $move(B, C)$ for a rank of 11, in agreement with the heuristic value $h_c^+(P[I = s, G = g])$. The interpretation that contains the actions $move(B, C)$ and $move(C, B)$ instead, is not a model of the theory, as it requires a justification of $at(B)$ in terms of $at(C)$ and a justification of $at(C)$ in terms of $at(B)$. The former implies $at(C) \prec at(B)$ and requires $\neg(at(B) \prec at(C))$, while the latter implies $at(B) \prec at(C)$ and requires $\neg(at(C) \prec at(B))$. The two justifications are thus inconsistent.

It is worth pointing out that while the heuristic h_c^+ and the optimal cost c^* coincide for this state, goal, and cost function, they do not coincide in general for other combinations. For example, if the goal g is to end up in the initial location $s = \{at(A)\}$ and the atom $at(C)$ is given cost -20 (i.e., a positive reward of 20), then the optimal plan is to go from A to C to collect the reward, and get back to A for a total cost of $c^*(P[I = s, G = g]) = -7$. The heuristic $h_c^+(P[I = s, G = g])$, on the other hand, is -9. The reason is that in the delete-relaxation the two actions $move(C, B)$ and $move(B, A)$ that are needed in order to get the agent back to $at(A)$ are not needed.

7. From heuristics to search

The last scenario illustrates an example in which heuristics and costs are both negative, and in which, even if the initial situation represents a goal state, the optimal plan is not empty. For the search of optimal plans, this implies that we cannot just plug the heuristic into an algorithm like A* and expect the algorithm to produce an optimal solution. Indeed, in the scenario above, the heuristic is admissible, the root node is a goal node, and yet the empty plan is not optimal. In order to use the heuristic h_c^+ to guide the search for plans in the PR model, we need to consider this issue as well.

We focus first on the use of the heuristic in a progression search from the initial state, and then briefly mention what needs to be changed for a regression search. First of all, in the PR model, a search node needs to keep track not only of the state of the system s but also of the set of fluents t with non-zero costs that have been achieved in the way to s. This is because penalties and rewards associated with such atoms are paid only once.[8] Thus, search nodes n must be pairs $\langle s, t\rangle$, and the heuristic $h(n)$ for those nodes must be set to

$$h(n) \stackrel{\text{def}}{=} h_{c_t}^+(s) \tag{13}$$

where $c_t(x) = c(x)$ for all actions and fluents x, except that $c_t(x) = 0$ if $x \in t$.

As in A*, the evaluation function $f(n)$ for a node n is set to the sum $g(n) + h(n)$ where $g(n)$ is the accumulated cost along the path $n_0, a_0, \ldots, a_i, n_{i+1}$ from the root n_0 to $n = n_{i+1}$

$$g(n) = c(n_0) + c(a_0, n_0) + c(a_1, n_1) + \cdots + c(a_i, n_i)$$

where

$$c(a_i, n_i) = c(a_i) + \sum_{\substack{p \in s_{i+1} \\ p \notin t_i}} c(p)$$

and

$$c(n_0) = \sum_{p \in s_0} c(p).$$

The root node n_0 of the search is the pair $\langle s_0, t_0\rangle$ where s_0 is the initial state and t_0 is the set of atoms $p \in s_0$ with non-zero costs, and node n_{i+1} is the pair $\langle s_{i+1}, t_{i+1}\rangle$ where s_{i+1} is the state that follows action a_i in the state s_i, and t_{i+1} is the union of t_i and the atoms in s_{i+1} with non-zero costs.

Due to the presence of negative heuristics and costs, the search algorithm cannot be the standard A* algorithm or Dijkstra. Yet a simple variant of the A* algorithm suffices provided that the heuristic is monotonic. Recall that a heuristic h is monotonic when the condition $h(n_i) \leqslant c(a_i, n_i) + h(n_{i+1})$ holds, a condition that ensures that the evaluation function $f(n) = g(n) + h(n)$ does not decrease along any search path [55]. The heuristic $h(n)$ defined by (13) is monotonic:

Proposition 8 *(Monotonicity of h_c^+). The heuristic $h(n) = h_{c_t}^+(s)$ where $n = \langle s, t\rangle$ is monotonic.*

In the revised A* algorithm, nodes n with minimum evaluation function $f(n)$ are selected iteratively from the OPEN list as in A*, but the loop does not terminate once a goal node is selected from OPEN. Rather the algorithm maintains the (accumulated) cost $g(n)$ of the best solution n found so far, and terminates when the cost of this solution is no greater than the evaluation function $f(n')$ of the best node in OPEN. It then returns n as the solution node. It is simple to show that this revised A* algorithm is optimal when the heuristic, like h_c^+, is monotonic even if the costs and heuristic have negative values.

[8] This implies that fluent penalties and rewards in the PR model are not markovian: an action a_i in a plan π that makes an atom p true contributes with a cost $c(p)$ to the cost of the plan π only if p has not been made true before. See [63] for a more general discussion of non-markovian rewards in the more general setting of Markov Decision Processes.

Proposition 9 *(A* with Negative Costs and Monotone Heuristics). The best-first search algorithm with the evaluation function of A*, $f(n) = g(n) + h(n)$, that terminates only when the cost $g(n)$ of the best solution node n found so far is no greater than the evaluation function $f(n')$ of the best node in OPEN, is optimal, even in the presence of negative edge costs, provided that the heuristic h is monotonic.*

If the heuristic h is monotonic, the evaluation function $f(n)$ will not decrease along any path from the root, and hence if a solution has been found with cost $g(n)$ which is no greater than $f(n')$ for every node in OPEN, then $g(n)$ will be no greater than the solutions that go through those nodes, and hence represents an optimal solution.

Unlike A*, the revised algorithm may terminate by reporting a node n in the CLOSED list as a solution. This happens for example when there are no goals but the heuristic $h(n_0)$ deems a certain reward worth the cost of obtaining it, when it is not. For example, if there is a fluent p with cost -10 such that the estimated and real cost for achieving it are 9 and 11 respectively, then the best plan is not to go for it and to do nothing for a cost of 0. However, initially $g(n_0) = 0$ and $h(n_0) = 9 + (-10) = -1 < 0$, and the cost of the best solution found so far, n_0 with a cost $g(n_0)$, is greater than the evaluation function $f(n_0) = g(n_0) + h(n_0) = 0 + (-1) = -1$ of the best (and only) node in OPEN. As a result the algorithm does not terminate, expands n_0, and keeps going until the best node n' in OPEN satisfies $f(n') \geqslant g(n_0) = 0$. At such a point, it returns n_0 as the solution node representing the empty plan. In [64], the termination condition of A* is also modified for dealing with (terminal) rewards (soft goals) but the proposed termination condition does not ensure optimality, as in particular, it will never report a solution node from the CLOSED list.

Most of this discussion carries directly to regression search where classical regression needs to be modified slightly: while in the classical setting, an action a can be used to regress a subgoal g when a 'adds' an atom p in g, in the penalties and reward setting, a can also be used when it adds an atom p, that while not in g, has a negative cost $c(p) < 0$.

8. Empirical results

We report some empirical results that illustrate the range of problems that can be handled using the proposed techniques. We derive the heuristic using the LP encodings and Theorem 7. The compilation into d-DNNF is done using

Table 1
Compilation data for logistic problems from 2nd IPC (serialized), some having plans with more than 40 actions. Time refers to compilation time in seconds, while size to the size of the resulting DAGs

Problem	Backward theory		Forward theory	
	time	size	time	size
4-0	0.1	6.2K	0.7	271K
4-1	0.1	6.2K	0.7	267K
4-2	0.1	5.9K	0.8	285K
5-0	0.1	5.9K	0.7	266K
5-1	0.1	5.9K	0.7	263K
5-2	0.1	6.2K	0.7	265K
6-0	0.1	5.9K	0.7	255K
6-1	0.1	5.9K	0.8	276K
6-2	0.1	6.2K	0.7	252K
6-3	0.1	6.2K	0.7	272K
7-0	0.7	24K	30.7	11M
7-1	0.7	23K	30.7	11M
8-0	0.7	24K	31.5	11M
8-1	0.7	24K	31.7	11M
9-0	0.7	24K	31.2	11M
9-1	0.7	24K	31.8	11M
10-0	3.3	94K	1603.7	434M
10-1	3.2	91K	1563.2	418M
11-0	3.2	91K	1597.7	427M
11-1	3.2	89K	1588.4	424M
12-0	3.3	94K	1578.3	418M
12-1	3.3	94K	1559.0	423M
13-0	1390.7	46M	–	–
13-1	1324.3	43M	–	–
14-0	1279.5	43M	–	–
14-1	1544.8	48M	–	–
15-0	1340.3	43M	–	–
15-1	1464.4	46M	–	–

Table 2
Search results for serialized logistics problems using the heuristics h^2 and h_c^+. For regression search, the heuristic h_c^+ is complemented with structural mutexes. The first column is the problem instance, the second the optimal cost (and length), and then three groups for the h^2 and h_c^+ heuristics, for backward and forward search, containing heuristic value at the initial state, number of expanded nodes and search time in seconds. A dash means the search did not finish within the limits of 2 hour and 2 Gb of memory

Problem	$c^*(P)$	h^2 backward			h_c^+ backward with mutex			h_c^+ forward		
		$h^2(P)$	nodes	time	$h_c^+(P)$	nodes	time	$h_c^+(P)$	nodes	time
4-0	20	12	4295	0.1	19	40	0.0	19	76	0.2
4-1	19	10	7079	0.1	17	109	0.0	17	259	0.8
4-2	15	10	537	0.0	13	25	0.0	13	72	0.2
5-0	27	12	118,389	4.0	25	490	0.0	25	1075	3.6
5-1	17	9	7904	0.2	15	103	0.0	15	212	0.6
5-2	8	4	143	0.0	8	8	0.0	8	8	0.0
6-0	25	10	316,175	13.1	23	668	0.1	23	932	3.0
6-1	14	9	1489	0.0	13	19	0.0	13	33	0.1
6-2	25	10	301,054	12.8	23	517	0.0	23	516	1.6
6-3	24	12	99,827	4.0	21	727	0.1	21	537	1.6
7-0	36	12	–	–	33	4973	4.6	33	10,693	5347.2
7-1	44	12	–	–	39	175,886	224.1	39	–	–
8-0	31	12	–	–	29	591	0.5	29	3685	1733.0
8-1	44	12	–	–	41	13,299	11.3	41	–	–
9-0	36	12	–	–	33	3083	2.6	33	14,088	6827.4
9-1	30	12	–	–	29	81	0.0	29	705	354.3
10-0	45	12	–	–	41	157,051	742.0	41	–	–
10-1	42	12	–	–	39	20,220	69.9	39	–	–
11-0	48	12	–	–	45	20,143	93.9	45	–	–
11-1	–	12	–	–	55	–	–	55	–	–
12-0	42	12	–	–	39	23,556	87.8	39	–	–
12-1	–	12	–	–	63	–	–	63	–	–
13-0	–	12	–	–	67	–	–	–	–	–
13-1	–	12	–	–	57	–	–	–	–	–
14-0	–	12	–	–	55	–	–	–	–	–
14-1	–	12	–	–	67	–	–	–	–	–
15-0	–	12	–	–	71	–	–	–	–	–
15-1	–	12	–	–	63	–	–	–	–	–

Darwiche's c2d compiler.[9] We actually consider a 'forward' theory used for guiding a progression search, and a 'backward' theory used for guiding a regression search. The first is obtained from the compilation of the formula $wffc(L(P)) \wedge G$ where G is the goal in P, while the second is obtained from the compilation of the formula $wffc(L(P)) \wedge I(s_0)$ where s_0 is the initial situation in P. This is because in a progression search the goal remains fixed throughout the search, while in a regression search the same is true for the initial situation. The heuristic h_c^+ for the regression search is complemented with structural 'mutex' information, meaning that the heuristic values associated with subgoals g that contain a pair of structurally mutex fluents are set to ∞. The mutex information is needed because a regression search tends to generate such impossible states which are not detected by heuristics that are based on the delete-relaxation [12]. All the experiments are carried out on a Linux machine with a Xeon processor running at 1.80 GHz with 2 Gb of RAM, and terminated after taking more than 2 hours or more than 2 Gb of memory.

Logistics. Table 1 shows the time taken by the compilation of some 'forward' and 'backward' logistic theories, along with the size of the resulting d-DNNF formula. These are all serialized instances from the 2nd International Planning Competition (IPC-2) [3], with several packages, cities, trucks, and airplanes, some having plans with more than 40 steps. Almost all of these instances compile, although backward theories, where the initial state is fixed, take much less time and yield much smaller representations. Table 2 provides information about the quality and effectiveness of the heuristic h_c^+ for the classical 0/1 cost function, in relation with the classical admissible heuristic h^2 [39], a generalization of the heuristic used in Graphplan [11]. The table shows the heuristic and real-cost values associated with the root nodes of the search, along with the time taken by a regression search and the number of nodes expanded. It can be seen that the h_c^+ heuristic is more informed than h^2 in this case, and scales up to problems that h^2 cannot solve.

It is important to emphasize that *once the theories are compiled they can be used to evaluate any state under any cost function*. So these logistics theories can be used for settings where, for example, packages have different priorities, loading them in various trucks involves different costs, etc. This applies to all domains.

[9] At http://reasoning.cs.ucla.edu/c2d.

Table 3
Search results for IPC-4 instances using the heuristics h^2 and h_c^+ in a regression search, the latest extended with structural mutexes. The first column reports the problem instance in each domain, the second, the optimal cost (equal to plan length in these problems), and the remaining columns, the value of the heuristic for the initial state, the number of nodes expanded, and the overall time. For h^2, a dash means that the search did not finish within the limits of 2 hours; while for h_c^+, either that the search (PSR-48) or the compilation (Airport-8, Airport-9; PSR-9; Satellite-4, Satellite-5, Satellite-6) did not finish

n	$c^*(P)$	h^2			h_c^+ backward with mutex		
		$h_2(P)$	nodes	time	$h_c^+(P)$	nodes	time
AIRPORT							
1	8	8	8	0.00	8	8	0.00
2	9	9	9	0.00	9	9	0.00
3	17	16	53	0.00	17	30	0.01
4	20	20	20	0.00	20	20	0.00
5	21	21	21	0.00	21	21	0.00
6	41	40	332	0.12	41	41	1.67
7	41	40	318	0.11	41	73	3.68
8	62	41	9412	8.74	–	–	–
9	71	41	57,549	69.55	–	–	–
PSR							
43	20	7	1263	0.04	4	1763	0.23
44	19	7	3446	0.17	5	3715	2.32
45	20	4	2033	0.06	4	2323	0.20
46	34	5	7,139,967	810.05	5	6,023,893	1595.02
47	27	4	32,863	1.72	4	31,219	3.82
48		8	–	–	5	–	–
49		9	–	–	–	–	–
50	23	4	26,249	1.27	6	17,927	3.69
SATELLITE							
1	9	7	15	0.00	8	20	0.16
2	13	7	559	0.02	12	110	38.36
3	11	6	1099	0.13	10	25	29.76
4	17	7	163,746	30.80	–	–	–
5		6	–	–	–	–	–
6		7	–	–	–	–	–

Blocks world. Blocks instances do not compile as well as logistic instances. We do not report actual figures as we managed to compile only the first 8 instances from the 2nd IPC. These are rather small instances having at most 6 blocks, where use of the heuristic h_c^+ does not pay off.

IPC problems. Tables 3 and 4 report the results of a regression search guided by the heuristic h^2 and by the heuristic h_c^+ over domains from IPC-4 and IPC-5 respectively; namely, Airport, PSR, Satellite, TPP, Pathways and Rovers. For the heuristic h^2, the dashes in the table mean that the search did not finish within the 2 hour time limit, while for h_c^+, that either the compilation did not finish (8 cases), or that the search did not finish (4 cases). Overall, the heuristic h^2 yields 2 more instances solved in the Airport domain and 1 in Satellite, while the heuristic h_c^+ yields 2 more instances solved in Rovers and 1 in TPP. In the other two domains shown, PSR and Pathways, the two heuristics solve the same number of problems. Usually, the heuristic h_c^+ ends up expanding less nodes but taking more time. The results of the compilation are not shown. Briefly, in most of the problems solved by the h_c^+ heuristic, the compilation finishes in less than a second, with 4 problems (Airport-6, Airport-7, PSR-48, Satellite-3) taking more than 2 minutes, and one problem (Satellite-3) taking more than 16 minutes. The computation of the heuristic h^2, on the other hand, is less expensive (although often less informed), taking more than 2 minutes only in two (solved) instances: Airport-6 and Airport-7. We also considered the Storage domain (from IPC-5 and not shown in the table), where the compilation for computing the h_c^+ heuristic works only for the 2 smallest instances. The heuristic h^2, on the other hand, yields solutions to the first 8 instances.

Elevator. The last domain consists of a building with n floors, m positions in each floor ordered linearly, and k elevators. There are no hard goals but various rewards and penalties associated with certain positions. All actions have cost 1. Fig. 1 shows the instance 10-5-1 with 10 floors, 5 positions per floor, and 1 elevator aligned at position 1 on the left. We consider also an instance 10-5-2 where there is an additional elevator on the right at position 5, and a larger instance involving 10 floors, 10 positions per floor and 2 elevators. The problem is modeled with actions for moving the elevators up and down one floor, for getting in and out the elevator, and for moving one unit, left or right, in each floor. These actions affect the fluents $at(f, p)$, $in(e, f)$ and $inside(e)$, where f, p and e denote a floor, a position, and an elevator respectively. The optimal plan for the instance 10-5-1 shown in Fig. 1 performs 11 steps to collect the rewards at floors 4 and 5 for a total cost of $11 - 14 = -3$. On the other hand, the instance 10-5-2 with another elevator on the right, performs 32 steps but obtains a better cost of -5. The LP encoding for computing the $h_c^+(P)$ heuristic doesn't compile for this domain except for very small instances. However, a good and admissible approximation $h_c^+(P')$ can be obtained by relaxing the problem P slightly by simply dropping the fluent $inside(e)$ from all the operators (this is a so-called pattern-database relaxation [19], where

Table 4
Search results for IPC-5 instances using the heuristics h^2 and h_c^+ in a regression search, the latest extended with structural mutexes. The first column reports the problem instance in each domain, the second, the optimal cost (equal to plan length in these problems), and the remaining columns, the value of the heuristic for the initial state, the number of nodes expanded, and the overall time. For h^2, a dash means that the search did not finish within the limits of 2 hours; while for h_c^+, either that the search (TPP-7, TPP-8; Rovers-6, Rovers-8) or the compilation (Pathways-5, Pathways-6) did not finish

n	$c^*(P)$	h^2			h_c^+ backward with mutex		
		$h_2(P)$	nodes	time	$h_c^+(P)$	nodes	time
TPP							
1	5	5	5	0.00	4	5	0.00
2	8	7	9	0.00	7	9	0.00
3	11	7	34	0.00	10	15	0.00
4	14	7	292	0.00	13	25	0.00
5	19	8	36,207	1.26	17	137	0.01
6	25	9	–	–	21	68,174	540.68
7		10	–	–	26	–	–
8		10	–	–	30	–	–
PATHWAYS							
1	6	6	6	0.00	6	6	0.00
2	12	10	47	0.00	12	12	0.01
3	18	11	9296	0.35	16	861	13.66
4	17	11	7969	0.32	15	327	38.61
5		11	–	–	–	–	–
6		13	–	–	–	–	–
ROVERS							
1	10	7	229	0.00	9	93	0.01
2	8	5	73	0.00	7	33	0.00
3	11	8	281	0.01	9	118	0.05
4	8	6	55	0.00	8	8	0.00
5	22	7	–	–	18	755,704	3118.57
6		8	–	–	27	–	–
7	18	6	–	–	15	118,937	3503.00
8		7	–	–	21	–	–

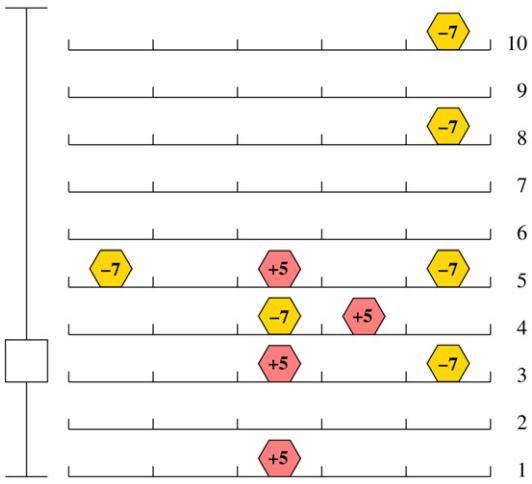


Fig. 1. Elevator instance 10-5-1 with 10 floors, 5 positions per floor, and 1 elevator at position 1. Penalties and rewards associated with the various positions shown. Instance 10-5-2 has a second elevator at position 5 (right most). The best plan for instance shown has length 11 and cost −3, while for 10-5-2, it has length 32 and cost −5.

certain atoms are dropped from the problem [29,37]). Using this technique, we were able to compile theories with up to 10 floors and 10 positions in less than a second. The problem in Fig. 1 is then solved optimally in milliseconds, expanding 65 nodes. As a reference, a 'blind' search based on the non-informative admissible heuristic h_0 that adds up all the uncollected rewards, takes 27 seconds and expands 445, 956 nodes. More results appear in Table 5 where it is shown that the heuristic is cost-effective in this case, and enables the optimal solution of problems that are not trivial.

Table 5
Search results for Elevator with h_0 and relaxed h_c^+ heuristics. Instance *n-m-k* refers to a problem with n floors, m positions and k elevators. Problem instance and its optimal cost are shown, and for h_0 and h_c^+, the value of the heuristic at the initial state, the length of the optimal plan, the number of expanded nodes, and the search time in seconds. A dash means that the search did not finish within the limits of 2 hours and 2 Gb of memory

Problem	$c^*(P)$	h_0 backward with mutex				relaxed h_c^+ backward with mutex			
		$h_0(P)$	len	nodes	time	relaxed $h_c^+(P)$	len	nodes	time
4-4-2	−1	−14	6	3399	0.0	−3	6	15	0.0
6-6-2	−6	−28	15	67,722	3.5	−13	15	499	0.0
6-6-3	−6	−28	15	472,446	31.7	−15	15	2505	0.6
10-5-1	−3	−42	11	445,956	27.0	−6	11	65	0.0
10-5-2	−5	−42	–	–	–	−21	32	66,486	16.7
10-10-2	−7	−63	–	–	–	−22	23	194,069	109.6

9. Boosting the heuristic: Plan constraints

The formulation of the delete-based relaxation heuristic in logic along with its computational model based on a compiled d-DNNF formula, can be extended in a natural way to a more powerful class of heuristics that are not bound by the limitations of the delete-relaxation. We consider one particular extension that results from taking *constraints on plans* into account: these are constraints over the sets of actions and fluents that are allowed in a plan: plans that violate such constraints will be ruled out by assigning them infinite cost. We will be interested in plan constraints that are valid in a problem in the sense that they are satisfied by some optimal plan. Making such constraints explicit has no effect on the *optimal cost of a problem* but will boost the *heuristic function* that is obtained from it by capturing information that is lost in the delete relaxation. For example, for suitable plan constraints, the new heuristic will be optimal for the Traveling Salesman Problem (TSP), where the delete-relaxation heuristic h_c^+, which is also exponential in the worst case (unless P = NP), yields the poorer Minimum Spanning Tree (MST) lower bound.

9.1. Syntax and semantics

A plan constraint C is a propositional formula over the sets of actions and fluents A and F. A plan π for a problem P satisfies a constraint C, written $\pi \models C$, if C is true over the interpretation that makes true only the actions and fluents in π and $F(\pi)$; i.e.

$$\begin{aligned}
&\pi \models C && \text{for } C \in A \cup F \text{ iff } C \in \pi \text{ or } C \in F(\pi),\\
&\pi \models \neg C && \text{iff } \pi \not\models C,\\
&\pi \models C \vee C' && \text{iff } \pi \models C \text{ or } \pi \models C',\\
&\pi \models C \wedge C' && \text{iff } \pi \models C \text{ and } \pi \models C'.
\end{aligned}$$

Intuitively, a constraint $p \vee q$ when p and q are fluents is satisfied by π when π makes p or q true at some point in the execution. Similarly, a constraint $\neg p \vee \neg q$ is satisfied when p, q, or both, are never made true by π. These plan constraints thus, should not be confused with the mutex constraints (p, q) [11] that express a constraint over the truth of p and q at the same time point. Plan constraints as defined are not as expressive as modal or temporal formulas, but are simple and sufficient for illustrating how the basic delete-relaxation heuristic can be improved.

Plan constraints are not used to modify the definition of plans but rather their *costs*, so that plans that do not comply with the constraints get an infinite cost:

Definition 10 *(Constrained Plan Costs).* The *constrained cost* $c(\pi, C)$ of a plan π for a planning problem P extended with a set of plan constraints C is

$$c(\pi, C) \stackrel{\text{def}}{=} \begin{cases} c(\pi) & \text{if } \pi \models C \\ \infty & \text{otherwise.} \end{cases} \tag{14}$$

Definition 11 *(Constrained Costs).* The *constrained optimal cost* $c^*(P, C)$ is

$$c^*(P, C) \stackrel{\text{def}}{=} \min_{\pi} c(\pi, C) \tag{15}$$

where π ranges over the plans for P, and $c^*(P, C) = \infty$ if no plan for P satisfies C.

Plan constraints increase the expressive power of the planning language, yet we are interested in plan constraints that are *implicit* in the sense that they have no effect on the cost of a problem:

Definition 12 *(Valid Plan Constraints).* A plan constraint C is *implicit* or *valid* in a problem P under a given cost function c when C is satisfied by some optimal plan if the problem admits a plan at all.

Clearly if C is a valid constraint for problem P under the cost function c, $c^*(P, C) = c^*(P)$. A sufficient condition for C to be valid for P under any cost function is when C is satisfied by all plans for P, whether optimal or not. For example, the constraint that prevents two moves away from the same city is true in all 'plans' that solve the Traveling Salesman Problem. On the other hand, the constraint that no block needs to be unstacked from two different blocks is a valid constraint in the Blocks World under the classical 0/1 cost function, but is not true in all plans and not even in all optimal plans (e.g., an optimal plan that moves a block to the table and then moves this block to its target destination can often be transformed into an optimal plan where the block is first moved on top of an irrelevant block instead).

The key point is that while valid plan constraints C do not affect the *optimal cost* of a problem P, they can potentially increase the value of the delete-relaxation *heuristic*:

Definition 13 *(Constrained Heuristic).* The *constrained* delete-relaxation heuristic of a planning problem P extended with the plan constraints C is defined as

$$h_c^+(P, C) \stackrel{\text{def}}{=} c^*(P^+, C) \tag{16}$$

where P^+ is the delete-relaxation of P.

The constrained delete-relaxation heuristic $h_c^+(P, C)$ can be more informed than the plain delete-relaxation heuristic $h_c^+(P)$ while remaining a lower bound on the true cost $c^*(P)$:

Theorem 14 *(Admissibility and Boosting). Let C be a plan constraint that is valid in P under the cost function c. Then,*

$$h_c^+(P) \leqslant h_c^+(P, C) \leqslant c^*(P, C) = c^*(P),$$

where the two inequalities can be strict.

The first inequality $h_c^+(P) \leqslant h_c^+(P, C)$ is direct and is always true as the second heuristic simply pushes the cost of some plans π in the relaxation P^+ to infinite, while the equality $c^*(P, C) = c^*(P)$ is true from the validity of C. The heuristic value can only increase from $h_c^+(P)$ to $h_c^+(P, C)$ when C is a constraint valid in P but invalid in the delete-relaxation P^+.

Theorem 14 is important as it says that the value of the delete-relaxation heuristic h_c^+ can be increased, while preserving admissibility, by simply making explicit certain valid but implicit plan constraints. Before showing how to account for the new heuristic $h_c^+(P, C)$ in the semantic and computational framework laid out above for $h_c^+(P)$, let us illustrate the difference between the two heuristics over a concrete example.

Consider a problem P where an agent, initially at location L, has to pick up a package p at location L' and return back to L with the package. If the available actions allow the agent to move from one location to the other, and pick up a package if at the same location as the package, then a plan like

$$move(L, L'), pick(p, L'), move(L', L)$$

is optimal for a cost $c^*(P) = 3$. The optimal cost of the relaxation P^+, on the other hand, is $c^*(P^+) = 2$ as in the absence of deletes, the last action $move(L', L)$ is not needed in order to return to L. This means that the delete-relaxation heuristic $h_c^+(P)$ is 2 as well.

Consider now the constraint $C : move(L, L') \supset move(L', L)$. This constraint is valid in P: if the agent moves away from the goal location then it has to get back to it. The delete-relaxation heuristic constrained with C can be shown to be $h_c^+(P, C) = c^*(P^+, C) = 3$, and hence strictly greater than the normal delete-relaxation heuristic $h_c^+(P) = c^*(P^+) = 2$. This is because while any plan for P^+ must include the actions $move(L, L')$ and $pick(p, L')$, the constraint C forces the action $move(L', L)$ to be in the plan too.

The example shows that the value of the delete-relaxation heuristic $h_c^+(P)$ can be boosted by making *explicit* certain constraints that are otherwise *implicit*. We do not consider in this paper the problem of deriving or learning such constraints automatically. Our goal instead is to show that such constraints can be naturally incorporated in the framework presented and that they can make a significant difference in relation to heuristics that are based on the delete-relaxation only.

9.2. Logical formulation and computation

Proposition 6 established the relation between the heuristic value $h_c^+(P[I = s, G = g])$, for any initial state s and goal g, and the rank $r^*(wffc(L(P)) \cup I(s) \cup g)$ of the propositional theory obtained from the logic program $L(P)$ encoding the relaxation P^+ of P, and the literals $I(s) \cup g$ encoding the state s and goal g. The extension of this correspondence in the presence of plan constraints C is straightforward:

Proposition 15 *(Constrained Heuristic and Ranks). For any initial situation s, goal g, cost function c, and plan constraint C,*

$$h_c^+\big(P[I=s, G=g], C\big) = r^*\big(wffc\big(L(P)\big) \cup C \cup I(s) \cup g\big)$$

where r is the literal ranking function such that $r(l) = c(l)$ for positive literals l and $r(l) = 0$ otherwise.

In other words, while the actions get mapped into logic programming rules that are then suitably closed, plan constraints get mapped into so-called *integrity constraints*: constraints that simply filter out some of the models [36]. From this result and the properties of formulas in d-DNNF, once again we obtain:

Theorem 16 *(Computing Constrained Heuristics). Let $\Pi(P, C)$ refer to the compilation of the theory $wffc(L(P)) \cup C$ into d-DNNF. Then* for any initial and goal situations *s* and *g, and any* cost function *c, the heuristic value $h_c^+(P[I=s, G=g], C)$ can be computed from $\Pi(P, C)$ in linear time.*

This means that in order to compute the *constrained* h_c^+ values, all we need to do is to add the plan constraints C to the CNF theory obtained from the logic program $L(P)$, before the CNF formula is compiled. Nothing else is needed. The differences in the resulting theory ranks, and hence, in the resulting heuristic values, can be quite significant however.

9.3. Examples

We will analyze the differences between the delete-relaxation heuristic $h_c^+(P)$ and the constrained heuristic $h_c^+(P, C)$ by considering two well known combinatorial problems: the Assignment Problem (AP), that is tractable, and the Traveling Salesman Problem (TSP), that is not. From a theoretical point of view, we will see that for simple but valid plan constraints C, the constrained heuristic $h_c^+(P, C)$ is optimal in these problems, i.e., equal to $c^*(P)$, while the delete-relaxation heuristic $h^+(P)$, which is also exponential in the worst case, is not. From an experimental point of view, we will see that the addition of implicit constraints, as it often happens in SAT, can help computationally as well.

The differences between the constrained and unconstrained heuristics $h_c^+(P)$ and $h_c^+(P, C)$ can be illustrated with the Sokoban-like problem shown in Fig. 2. In this problem, the stones in the grid (shown as diamonds) must be moved to the goal cells (shown with the letter 'G') by means of actions $move(t, l, l')$ that move a stone t from location l into a free location l' at a cost that is equal to the Manhattan Distance between l and l'. This problem is simpler than Sokoban yet no existing domain-independent heuristic that we are aware of captures the actual cost of the problem. For the instance shown in the figure, the optimal cost is $c^*(P) = 16$, while the delete-relaxation heuristic is $h_c^+(P) = 6$. The under-estimation results from assuming that the cells that are initially free, remain free, ignoring thus that no two stones can end up in the same goal cell. This constraint, however, can be enforced in the constrained heuristic $h_c^+(P, C)$ by making C stand for the valid plan constraint that disallows pairs of actions $move(t_1, l_1, l')$ and $move(t_2, l_2, l')$ that move two different stones t_1 and t_2 into the same cell l' for any l'. Provided that plans containing such pairs of actions are disallowed in the relaxation, the best plans have a cost of 16, and thus, the constrained heuristic is $h_c^+(P, C) = 16$, like the optimal problem cost.

9.3.1. Assignment problem

The heuristic $h^+(P, C)$ above is similar to the heuristic used in the specialized, optimal Sokoban solver described in [42] which is based on the Assignment Problem [56]; this is the problem of finding an injective mapping $f : X \to Y$, i.e., from elements of X into distinct elements in Y, such that the cost $\sum_{x \in X} cost(x, f(x))$ is minimized for a given positive cost function $cost(x, y)$ over all $x \in X$ and $y \in Y$, that without loss of generality we assume to be positive (if the cost function is not positive, a positive increment can be added to all pairs without affecting the solutions). Our simplified version of Sokoban is an Assignment Problem where X is the set of stones, Y is the set of goal locations, and $cost(x, y)$ is the Manhattan Distance between the current location of stone x and y.

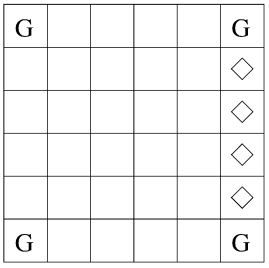


Fig. 2. A Sokoban-like problem with 4 stones to be moved to the four corners with actions $move(t, l, l')$ that move stone t from location l to a free location l' at a cost given by the Manhattan Distance between l and l'. The optimal cost is $c^*(P) = 16$, while the heuristics are $h_c^+(P) = 6$ and $h_c^+(P, C) = 16$, where C is the valid constraint $\neg move(t_1, l_1, l) \vee \neg move(t_2, l_2, l)$ that prevents moving two stones into the same cell.

Any such assignment problem can be formulated into a planning problem through an encoding with fluents

$$\mathit{assigned}(x), \quad \mathit{free}(y)$$

for each $x \in X$ and $y \in Y$, and actions $\mathit{map}(x, y)$ with precondition, add, and delete lists

$$P : \mathit{free}(y); \quad A : \mathit{assigned}(x); \quad D : \mathit{free}(y).$$

Initially, $\mathit{free}(y)$ is true for all $y \in Y$, while $\mathit{assigned}(x)$ must be true for all $x \in X$ in the goal.

It is easy to show that each plan of the resulting encoding corresponds to an assignment and that, for the cost function $c(\mathit{map}(x, y)) = \mathit{cost}(x, y)$, the optimal plans corresponds to the optimal assignments. Furthermore, by making explicit the *valid* plan constraint C

$$\neg \mathit{map}(x, y) \vee \neg \mathit{map}(x', y)$$

for all $y \in Y$ and $x, x' \in X$ such that $x \neq x'$ (not two x's mapped into the same y), the heuristic $h^+_c(P, C)$ is optimal.

Theorem 17 *(Assignment Problem). Let P be the planning problem encoding an arbitrary* assignment problem*, and let C be the valid plan constraint that prevents assigning two domain elements to the same target element. Then, the heuristic $h^+_c(P, C)$ is optimal*; *i.e., $h^+_c(P, C)$ is equal to the cost of the optimal assignment.*

The delete-relaxation heuristic $h^+_c(P)$, on the other hand, is not optimal and may not even reflect the cost of any assignment. Indeed, while $h^+_c(P, C)$ yields the minimum of $\sum_{x \in X} \mathit{cost}(x, f(x))$ over all the injective (one-to-one) functions $f : X \to Y$, $h^+_c(P)$ yields the minimum over all functions $f : X \to Y$, injective or not.

This class of problems shows that valid plan constraints can be used for capturing structural information that is lost in the delete-relaxation without the need for bringing time indices back in the formulation. We will show that the same happens in another combinatorial problem that unlike the Assignment Problem is intractable.

9.3.2. The Traveling Salesman problem

We consider now a problem harder than the Assignment Problem: the Traveling Salesman Problem (TSP), a well-known combinatorial problem that surfaces in a number of applications and is known to be intractable [56]. Once again we will encode the TSP as a planning problem, and prove that for simple valid plan constraints C, the constrained heuristic $h^+_c(P, C)$ is optimal while the unconstrained heuristic $h^+_c(P)$, which is also exponential in the worst case, is not. Moreover, we will show that the latter captures the Minimum Spanning Tree (MST) lower bound, which unlike the TSP can be solved efficiently [56].

The TSP is the problem of finding a minimum cost tour in a given directed graph $G = (V, E)$ extended with cost information $c(x, y)$ on the edges $(x, y) \in E$ which we assume to be positive (else positive increments can be added to all edges without affecting the solutions). A tour is a path that starts and ends in the same vertex, visiting all other vertices in the graph exactly once. The TSP problem can be encoded as a planning problem with the fluents

$$\mathit{at}(x), \quad \mathit{visited}(x), \quad \mathit{not\text{-}visited}(x),$$

for $x \in V'$, where $V' = V \cup \{x_f\}$ and x_f is an additional, dummy vertex that stands for a copy of the first vertex x_0 in the tour, chosen arbitrarily from V. The actions $\mathit{move}(x, y)$ for $x \in V$, $y \in V'$ and $x \neq y$, have precondition, add, and delete lists

$$P : \mathit{at}(x), \mathit{not\text{-}visited}(y); \quad A : \mathit{at}(y), \mathit{visited}(y); \quad D : \mathit{at}(x), \mathit{not\text{-}visited}(y).$$

The action costs $c(\mathit{move}(x, y))$ must be set to $\mathit{cost}(x, y)$ if $(x, y) \in E$ and $y \neq x_f$, to $\mathit{cost}(x, x_0)$ if $y = x_f$ and $(x, x_0) \in E$, to 0 if $x = x_0$ and $y = x_f$, and to ∞ otherwise. If the graph is sparse a more compact encoding results from using the set of edges E to exclude actions $\mathit{move}(x, y)$ when (x, y) is not in E. The theoretical results, however, apply to either encoding. Finally, the initial situation must be set to the atoms $\mathit{at}(x_0)$, $\mathit{visited}(x_0)$, $\mathit{not\text{-}visited}(y)$ for all $y \in V'$, $y \neq x_0$, while the goal situation must be set to $\mathit{visited}(y)$ for all $y \in V'$.

It is easy to show that this encoding is correct; namely that if $(x_0, x_1, \ldots, x_n, x_0)$ is an optimal solution of a TSP, the action sequence $\langle \mathit{move}(x_0, x_1) \ldots, \mathit{move}(x_n, x_f) \rangle$ is an optimal solution with the same cost in the corresponding planning problem. More interestingly, we can prove that the heuristic $h^+_c(P, C)$ is *optimal* provided that C is the constraint that rules out two moves away from the same vertex:

Theorem 18 *(Optimality of Constrained h^+_c for TSP). Let T be a TSP, P and c stand for its planning encoding, and let C stand for the valid plan constraint*

$$\neg \mathit{move}(x, y) \ \vee \ \neg \mathit{move}(x, y')$$

over the actions in P for all x and $y \neq y'$. Then, the heuristic $h^+_c(P, C)$ is optimal; *i.e., $h^+_c(P, C)$ is equal to the cost of the optimal TSP solution.*

This result reflects a correspondence between the plans for the delete-relaxation P^+ that comply with the constraints C, and the TSP tours. In the absence of C, however, the delete-relaxation heuristic, while also intractable in the worst case, captures only the tractable Minimum Spanning Tree bound.

A Spanning Tree of a (directed/undirected) graph $G = (V, E)$ is (directed/undirected) subgraph $G' = (V, E')$ that includes all the vertices V and a subset E' of the edges E defining a (directed/undirected) tree over V [18,56]. A Spanning Tree has minimum cost and hence is a Minimum Spanning Tree of G when the sum of the costs associated with the selected edges is minimized. For a directed graph $G = (V, E)$ with a distinguished root vertex $v \in V$, its MST is a Spanning Tree with minimum cost rooted at v. Finding a MST for directed or undirected graphs can be done efficiently [32]. Also, the cost of the Minimum Spanning Tree of G is a lower bound on the cost of the optimal TSP tour with respect to any root vertex, as the path that results from removing any edge from the tour is a Spanning Tree for G.

While the heuristic $h^+_C(P, C)$ in Theorem 18 captures exactly the cost of optimal TSP tour of a graph G, the heuristic $h^+_C(P)$ based exclusively on the delete-relaxation, captures the cost of the MST for G rooted at the location x_0 selected as the initial location in the planning encoding:

Theorem 19 *(Unconstrained h^+_C and MST Lower Bound). Let T be a TSP, and let P and c stand for its planning encoding. Then, the value of the unconstrained delete-relaxation heuristic $h^+_C(P)$ is equal to the cost of the Minimum Spanning Tree of T rooted at the location x_0 selected as the initial location in P.*

9.4. Experiments with plan constraints

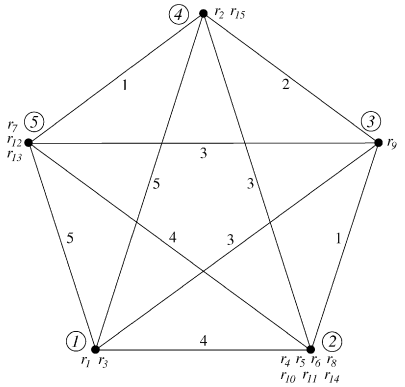
We have seen above that making explicit certain valid plan constraints can lead to an improved heuristic where structural information that is lost in the delete-relaxation is recaptured. Here we aim to test the computation and the use of the new heuristic empirically. We will see that the addition of these constraints yields indeed more accurate bounds but without necessarily making the computation more expensive.

We consider a variant of the TSP problem in which there are a number of rocks of different classes at various locations, and the goal is to analyze a rock from each class. The optimal plan thus involves a minimum cost path that visits a set of locations that contain rocks of all classes but does not have to visit all locations. The problem combines two intractable tasks whose solutions are coupled: the selection of the rock in each class to visit (Set Cover), and the selection of the path to visit them (TSP). Later on we consider a variation that adds another level of complexity, where the hard goal of having rocks of all classes analyzed is replaced by soft rewards so that the classes of rocks that are worth analyzing must be selected too. We refer to the former as the 'hard' version of the Rock Analyst problem, and the latter, as the 'soft' version. This domain is a variation of a domain considered in [60]. Fig. 3 displays an instance of the problem with 5 locations and 15 rocks of 10 different classes where the agent is initially at location 1. The classification of the rocks in the 10 classes, as well as the rewards in the soft version of the problem, are shown on the right.

For modeling the problem, we extend the planning model of the TSP with the fluents $type(r, c)$, $contains(x, r)$ and $analyzed(c)$ that specify the class and location of a rock and whether a rock of a given class has been analyzed, and actions $analyze(r, x, c)$ with precondition, add, and delete lists

$$P : at(x), contains(x, r), type(r, c); \quad A : analyzed(c); \quad D : --$$

that are used to analyze a rock r of class c located at x. In the hard version of the problem, the goal is to have rocks of all classes analyzed, and thus the goal includes the atoms $analyzed(c)$ for each class c. In the soft version of the problem these



rock	class	reward
r_1	c_1	-2
r_2	c_2	-5
r_3	c_3	-1
r_4	c_4	-5
r_5	c_5	-5
r_6	c_6	-3
r_7	c_7	-2
r_8	c_8	-5
r_9	c_9	-5
r_{10}	c_{10}	-3
r_{11}	c_4	
r_{12}	c_2	
r_{13}	c_6	
r_{14}	c_3	
r_{15}	c_3	

Fig. 3. Rock Analyst instance with 5 locations and 15 rocks classified into 10 classes ($n = 5$ in the tables). The map shows the locations, the rocks in each location, and the edge costs. The class of each rock and the rewards used in the 'soft' version of the problem are shown on the right. Location 1 is the initial location. An optimal plan for the hard and soft versions of the problem, involves the path ① → ③ → ② → ⑤, where the rocks $r_1, r_9, r_5, r_8, r_{10}, r_{11}, r_{14}, r_7, r_{12}, r_{13}$ are analyzed, for a total cost of 18 and −18 respectively.

Table 6
Compilation results for Rock Analyst with hard goals: problem instance and optimal costs shown along with the value of the heuristic at the initial state, and the time to compute the heuristic h^2 and h_c^+ with and without the constraint C that rules out two moves away from the same location. For the heuristics h_c^+, the size of the compilation in bytes is also shown. A dash means the theory did not compile within the limits of 1 hour and 1 Gb of memory

n	$c^*(P)$	h^2		h_c^+ unconstrained			h_c^+ constrained		
		$h^2(P)$	time	$h_c^+(P)$	time	size	$h_c^+(P,C)$	time	size
4	13	7	0.2	12	0.0	2.2K	13	0.0	3.2K
5	18	10	0.8	17	0.0	9.2K	18	0.0	12K
6	18	6	2.1	18	0.1	56K	18	0.2	65K
7	22	8	5.0	21	1.3	587K	22	1.4	392K
8	23	7	10.6	22	18.3	6.3M	23	16.3	3.9M
9	26	8	20.7	25	367.6	73M	26	1217.7	22M
10	29	7	37.9	–	–	–	–	–	–
11	31	7	65.9	–	–	–	–	–	–
12	–	7	107.6	–	–	–	–	–	–

atoms are given positive rewards (negative costs). We consider random problems with n locations, $2n$ non-empty classes, and $3n$ rocks, for $n = 4, 5, \ldots, 12$.

Table 6 shows the compilation results for the 'hard' version of the problem where rocks of all classes must be analyzed. The table displays the costs of the optimal tour, the values of the heuristics $h_c^+(P)$ and $h_c^+(P, C)$, where C is the constraint that prevents two moves away from the same location,[10] and the time and size of the compilation in d-DNNF. In addition, as a reference, the value of the h^2 heuristic [39], along with the time for computing the heuristic for all atom pairs is reported too. As it can be seen from the table, the theory with constraints results in optimal heuristic values $h_c^+(P, C) = c^*(P)$, while the theory without constraints gives lower bounds $h_c^+(P) \leqslant c^*(P)$, which in these instances are quite accurate. The compilation of the theory with and without constraints scales up only up to the problem with the number of locations $n = 9$, with the former producing smaller d-DNNF formula although not always in less time. The heuristic h^2, on the other hand, is quadratic and scales up to larger problems, although is not well informed, and results in a search that explores an exponentially larger number of nodes.

Table 7 shows the results of a regression search guided by these three heuristics. The search algorithm, as before, is A* with the revised termination condition that ensures optimality in the presence of negative costs and a monotone heuristic.[11] The three heuristics are extended with structurally mutex information so that states in the regression that include an unreachable atom pair are immediately pruned. Not surprisingly, the search with the constrained heuristic $h_c^+(P[G = g], C)$ yields the best results with the least number of expanded nodes. Yet, once the compilation effort is taken into account, the standard h^2 heuristic is best, managing to solve the instances with $n = 10$ and $n = 11$ that the other two heuristics cannot solve, even if they expand a much lower number of nodes. In each of the cases, the table shows also the lengths of the optimal plans found; all these plans must have the same (optimal) cost but do not have to have the same length. It should be noted that while the heuristic $h_c^+(P[G = g], C)$ is optimal in this problem when g is the *top goal*, it is not necessarily optimal over all the subgoals g' that arise in the regression. This explains why the number of nodes expanded by the heuristic in the regression is sometimes larger than the number of actions in the optimal plan.

We also considered a 'soft' version of the problem where the fluents *analyzed*(c) are removed from the goal and are given rewards (negative costs) randomly chosen between $[-6, -1]$. Table 8 shows the results for the compilation. Once again, the constraints improve the size of the compilation, although not necessarily the time. The heuristic used as a reference is the heuristic h_0 that is also admissible and simply adds up all the uncollected rewards. The heuristic values $h_0(P)$ are much less informed than $h_c^+(P)$ and $h_c^+(P, C)$ but much cheaper to compute.

Table 9 shows the results of the search. In this case, $h_c^+(P)$ and $h_c^+(P, C)$ do best, with the latter expanding a smaller number of nodes in less time. The heuristic h_0, as the heuristic h^2 in the 'hard' version of the problem, requires an exponentially larger number of nodes to be explored scaling up to problems with $n = 7$. The other two heuristics scale up with the compilation until $n = 9$.

[10] This constraint is valid provided that all locations are connected and that the direct path between any two locations is not more costly than an indirect path. These two conditions are true in the problems below.

[11] The search for optimal plans using the heuristic function h_c^+ extended with a set of constraints C raises two additional issues that need to be addressed that we mention briefly here. First, for the heuristic to remain consistent during the search with constraints, search nodes must become triplets $n = \langle s, t, v\rangle$, where s and t stand as before for the progressed state and the set of atoms p encountered in the path to n from the root with cost $c(p) \neq 0$, and the new component v is the set of atoms p encountered along the same path that occur in C. The heuristic $h(n)$ of the node n is then $h_{c_t}^+(P[I = s], C_v)$, where c_t is a cost function like c but with $c(p) = 0$ if $p \in t$, and C_v is a constraint like C but with p replaced by the boolean *true* (a similar representation is needed for a regression search). Second, since the compilation of the theory $wffc(L(P)) \cup C$ ensures that the heuristic values $h_c^+(P[I = s, G = g], C)$ can be computed in linear time for any state s, subgoal g, and cost function c, but not for any constraint C, in order to compute $h(n) = h_{c_t}^+(P[I = s], C_v)$ in linear time for any $n = \langle s, t, v\rangle$, a theory $wffc(L(P)) \cup C'$ slightly different than $wffc(L(P)) \cup C$ needs to be compiled, where C' is C but with all its variables x replaced by copies x', along with the implications $x \supset x'$. We omit the details and proofs of these extensions that are implemented in the planner.

Table 7
Search results for Rock Analyst with hard goals: problem instance and optimal cost shown, along with the plan length, number of nodes expanded, and plan time in seconds, for the heuristics h^2 and h_c^+ with and without the constraint C. A dash means that the search did not finish within the limits of 1 hour and 2 Gb of memory

n	$c^*(P)$	h^2			h^+ unconstrained			h_c^+ constrained		
		len	nodes	time	len	nodes	time	len	nodes	time
4	13	11	190	0.0	11	43	0.0	11	12	0.0
5	18	14	1982	0.0	14	161	0.0	13	13	0.0
6	18	16	5901	0.2	16	191	0.0	16	79	0.0
7	22	19	26,548	1.6	19	283	0.9	19	19	0.0
8	23	22	75,758	5.7	22	456	43.2	22	38	3.6
9	26	24	452,155	46.3	23	667	729.2	23	23	18.8
10	29	27	1,612,766	197.6	–	–	–	–	–	–
11	31	30	4,213,661	620.8	–	–	–	–	–	–
12	–	–	–	–	–	–	–	–	–	–

Table 8
Compilation results for Rock Analyst with soft goals: problem instance and optimal cost shown along with the value of the heuristic at the initial state with the time in seconds needed to compute the heuristic h_0 (sum of uncollected rewards) and h_c^+ with and without the valid constraint C that rules two moves away from the same location. For the heuristics h_c^+, the size of the compilation in bytes is also shown. A dash means the theory did not compile within the limits of 1 hour and 1 Gb of memory

n	c^*	h_0		h_c^+ unconstrained			h_c^+ constrained		
		$h_0(s_0)$	time	$h_c^+(P)$	time	size	$h_c^+(P, C)$	time	size
4	−5	−18	0.0	−6	0.0	2.2K	−5	0.0	3.2K
5	−18	−36	0.0	−19	0.0	9.2K	−18	0.0	12K
6	−19	−37	0.0	−19	0.1	56K	−19	0.2	65K
7	−19	−40	0.0	−20	1.3	587K	−19	1.4	392K
8	−25	−48	0.0	−26	18.2	6.3M	−25	16.4	3.9M
9	−19	−44	0.0	−19	366.7	73M	−19	910.3	23M
10	–	−54	0.0	–	–	–	–	–	–
11	–	−76	0.0	–	–	–	–	–	–
12	–	−64	0.0	–	–	–	–	–	–

Table 9
Search results for Rock Analyst with soft goals: problem instance and optimal cost shown, along with the plan length, number of nodes expanded, and plan time in seconds, for the heuristics h_0 and h_c^+ with and without the constraint C. A dash means that the search did not finish within the limits of 1 hour and 2 Gb of memory

n	$c^*(P)$	h_0			h_c^+ unconstrained			h_c^+ constrained		
		len	nodes	time	len	nodes	time	len	nodes	time
4	−5	4	7959	0.3	8	53	0.0	4	5	0.0
5	−18	11	119,729	7.3	14	168	0.0	13	13	0.0
6	−19	13	1,461,436	168.9	14	461	0.3	14	77	0.1
7	−19	12	8,175,484	1363.7	17	131	1.2	17	33	0.2
8	−25	–	–	–	21	1653	262.5	20	36	7.9
9	−19	–	–	–	19	168	611.4	19	19	35.3
10	–	–	–	–	–	–	–	–	–	–
11	–	–	–	–	–	–	–	–	–	–
12	–	–	–	–	–	–	–	–	–	–

10. Discussion

In this work we have used propositional logic in various forms (CNF, Logic Programs, d-DNNF), for defining admissible heuristic for planning in the presence of costs and rewards that are computed by means of circuits that map situations into values in time that is linear in the circuit size. This circuit is a simple transformation of the d-DNNF formula encoding the domain where ANDs and ORs are replaced by Adders and Min Operators, and whose output for any situation encodes the cost of the best model. In the worst case, the d-DNNF formula, and hence the evaluation circuit, can have a size that is exponential in the treewidth of the CNF encoding [21]. While this is a worst-case bound and the experiments show that a number of non-trivial problems can be solved by heuristics defined and computed in this way, it nonetheless expresses the practical limitation of the formulation in its current form. A possible way to overcome this limitation is the use of additional relaxations able to map CNF theories into weaker theories of bounded treewidth that can be then compiled in polynomial time and space. This is the approach we have taken recently in [58] for solving MinCostSAT problems. This type of relaxation in a planning setting results in a family of tractable, admissible heuristics, computable by circuits, that capture information about the delete lists in the problem in the form of valid plan constraints. Preliminary experiments using these heuristics are reported in [57].

The effective use of propositional logic in classical planning has been championed by Kautz and Selman in [43–45], where Strips problems P are mapped into CNF theories $T(n)$ for a planning horizon n that are fed into state-of-the-art SAT solvers. The encoding ensures a correspondence between the logical models of $T(n)$ and the plans for P. A similar approach but in the context of Logic Programming, has been proposed in [27,46,62] where CNF encodings of planning problems are replaced by Logic Programs whose Answer Sets, computed by state-of-the-art ASP Solvers, are in correspondence with the target plans. Logic programs allow for more concise encodings although this does not translate necessarily into faster solutions due to speed of current SAT solvers.

The differences between our use of logic for planning and these other approaches are basically three. First, we use logic for defining and computing heuristics for planning, not for computing the plans themselves. This allows us to use more compact encodings that do not require time indices or planning horizons. This is important because, the complexity of the encodings grows exponentially with the horizon, and except for the plan metric defined by the horizon, the optimality of the plans obtained is conditional on the horizon used.

Second, the resulting logical encoding is not solved using a SAT or ASP solver, but using a d-DNNF compiler. Once the problem is compiled, the heuristic values for any search state are found in time that is linear in the compiled representation. Incidentally, the d-DNNF compiler can be thought of as a SAT solver with a suitable form of caching that computes all solutions and leaves a 'trace' behind that enables the solution of other problems as well [38]. Another use of d-DNNF in planning is reported in [54], where heuristics for planning under incomplete information and no sensing (conformant planning) are obtained using projections and model counts computable in linear-time over d-DNNF representations [25].

Third, the heuristics handle a richer cost structure where actions can have non-uniform costs, and penalties and rewards may be associated with fluents. The computation of optimal plans in rich cost structures has been pursued recently in [8, 30,35] where planning problems are mapped into optimization variants of SAT, Answer Sets, or CSPs (Constraint Satisfaction Problems) like Weighted Max-SAT, Weighted ASP, and Weighted CSPs. A serious limitation of these approaches, however, is that the optimality of the resulting plans is conditional on the horizon used. Moreover, increasing the planning horizon one by one is no alternative because it is not clear when to stop, as plans with a larger makespan can potentially decrease the overall cost.

Approaches for planning with preferences in the form of soft goals are reported in [7,60,64]. The latter two approaches, however, are not optimal for various reasons: in some cases, heuristic mechanisms are used to select the soft goals to pursue, in others, a fixed planning horizon is assumed, and finally in others, a suboptimal Anytime A* search algorithm is used. The work in [7], on the other hand, presents an optimal planner for dealing with temporally extended preferences based on a branch-and-bound search, yet the lower bounds utilized are not sufficiently informed, and thus the search cannot be run to completion in general.

The problems of selecting a subset of the soft goals to pursue along with the estimation of the cost for achieving them are tightly coupled. The heuristics developed in this paper tackle them both at the same time: the best model of the theory selects the 'soft goals' that are to be pursued at the same time that it estimates the cost of achieving them. The heuristic is indeed optimal for this task when there are no deletes.

11. Summary

We have combined ideas from a number of areas, including search, planning, knowledge compilation, and answer set programming to develop

1. a cost model for planning that accommodates fluent penalties and rewards,
2. a generalization of the admissible delete-relaxation heuristic h_c^+ for informing the search in the model,
3. an account of the heuristic in terms of the rank of the best models of a suitable propositional theory obtained from the strong completion of a logic program that does not require time indices or horizons,
4. an approach that exploits this formulation along with the properties of d-DNNF formula for computing all heuristic values h_c^+ needed in the search in time linear in the size of the compilation,
5. a best-first algorithm able to use this heuristic, capable of handling negative heuristics and costs while ensuring optimality, and
6. an extension of the framework based on the use of valid plan constraints for boosting the values of the heuristic, overcoming some of the limitations of the delete-based relaxation.

We have also analyzed some of the properties of the resulting heuristics such as admissibility and monotonicity, as well as some of the semantic differences between the heuristic $h_c^+(P)$ and constrained heuristic $h_c^+(P, C)$, showing for example that the latter produces optimal costs in problems such as the Traveling Salesman Problem where the former yields only the Minimum Spanning Tree lower bound, even if both heuristics are exponential in the worst case. We have also implemented these ideas in a planner that we tested on a number of problems, producing non-trivial results, including the best results we are aware of in some domains.

The computational bottleneck in this approach is the compilation of the CNF translation of the logic program into a d-DNNF formula. In [58] and [57], we report preliminary results of an additional relaxation method that maps the CNF formula into a weaker formula of bounded treewidth that can be compiled efficiently.

The results above do not have to be taken as a single package, and in particular it is possible to replace the compilation step by an explicit computation of the best models for each of the states that arise in the search, using a Weighted SAT or ASP solver. The appeal of the compilation-based approach, however, is that it yields what can be deemed as a *circuit* or *evaluation network* whose input is a situation and whose output, produced in linear-time, is an appraisal of the situation in the form of its heuristic value. Some authors have associated evaluations of this type with the role played by emotions in real agents [20,28].

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Appendix A. Proofs

We include proofs of the results in the paper whose derivation is not direct from the text.

Proof of Proposition 1. Any plan π for P is a plan for the relaxation P^+ as well, and if $F(\pi)$ and $F^+(\pi)$ stand for the set of atoms made true by π in P and in P^+ respectively, clearly $F(\pi) = F^+(\pi)$. It follows then that any plan π in P is a plan for the relaxation P^+ with the same cost, and hence that $h^+_c(P) = c^*(P^+)$ cannot be higher than $c^*(P)$. □

Proof of Proposition 2. The correspondence between the plans of length n and the models of $T_n(P)$ for a sufficiently large n is established in [44]. In the worst case, this horizon can be exponential [15]. □

Proof of Proposition 4. The proposition follows from the correspondence between the models of $T^+_n(P) \cup I_0 \cup G_n$ and the plans for $P^+[I = s, G = g]$. The models M of $T^+_n(P) \cup I_0 \cup G_n$ are in 1–1 correspondence with the plans π for $P^+[I = s, G = g]$ for a sufficiently large n. Moreover, for the literal-ranking function given in the proposition, $r(M) = c(\pi)$, as if the action a occurs at time i in the plan, then a_i is true in the model, and hence the contribution of this action to $c(\pi)$ and $r(M)$ is the same. In addition, if a fluent p is made true by the plan at any one point, due to the absence of deletes, p must be true too at the end of the plan, and hence p_n must be true in the model. Since $r(p_n) = c(p)$, fluents also contribute the same terms to both $c(\pi)$ and $r(M)$. □

Proof of Proposition 6. We establish a correspondence between the plans π for the $P^+([I = s, G = g])$ and the models of $wffc(L(P)) \cup I(s) \cup g$. An interpretation over the theory $T = wffc(L(P)) \cup I(s) \cup g$ can be expressed as a pair (A, F) of actions A and fluents F. Moreover, in the models (A, F) of $wffc(L(P))$, the set of fluents F is determined by the set of actions A as from the definition of $wffc(L(P))$, F is the unique minimal model of the logic program $L(P) \cup A$. The set of fluents F that are true in such a model, that we call $F(A)$, can be stratified into sets $F_0(A)$, $F_1(A)$, ..., where $F_0(A)$ contains the heads p of rules $p \leftarrow Pre(a), a$ in $L(P)$ such that $a \in A$ and $Pre(a)$ is empty, while $F_{i+1}(A)$ contains the heads of the same rules with $a \in A$ and $Pre(a)$ becoming true at i ($Pre(a)$ becomes true at i if some $q \in Pre(a)$ is in $F_i(A)$, and the other atoms $r \in Pre(a)$, if any, are in $F_k(A)$, for $k \leqslant i$).

Now, if (A, F) is a best model of T such that for some actions $a \in A$, their preconditions are not all true in F, by construction of $L(P)$ and since $c(a) \geqslant 0$ it is possible to construct another model (A', F) of T, $A' \subset A$, with these actions eliminated and with the same set of fluents, that is as good as (A, F) and where all preconditions are satisfied. We want to show that such a model (A', F) encodes a 'parallel' plan π for $P^+([I = s, G = g])$ with the same cost. In the delete-relaxation, any parallel Strips plan can be serialized, keeping its cost, and also, any sequential plan can be parallelized by moving the actions to the first time point where its preconditions hold. The parallel plan π that corresponds to (A', F) is defined as the sequence of action sets $A_0, A_1, \ldots$ where A_i is the set of actions in A' whose preconditions become true at i in $F(A')$, with the 'set actions' $set(p)$ excluded. Clearly, π is a parallel plan that maps $I = s$ into $G = g$ as the preconditions of each action in the plan are either true in s or are added by an earlier action (that no action deletes as P^+ is delete-free), and it must include g as it 'adds' the same fluents as (A', F) that is a model of g. This shows that from a model (A, F) of $wffc(L(P)) \cup I(s) \cup g$ we can go to a plan for $P^+([I = s, G = g])$ with the same cost.

We are left to show the converse: that plans π for the $P^+([I = s, G = g])$ translate into models of $wffc(L(P)) \cup I(s) \cup g$ with the same cost. For this we need to show that the interpretation (A, F) where A contains all the actions in π as well as the 'set' actions in $I(s)$, and F is the set of atoms added by these actions is a model of $wffc(L(P)) \cup I(s) \cup g$, or equivalently, that (A, F) is a minimal model of the program $L(P) \cup A \cup I(s)$ that satisfies g. The latter is clear as g, being the goal, must be added by an action in π and hence must be part of F. For the former, it must be shown that the fluents p in F can be partitioned into sets F_0, F_1, ..., such that $p \in F_0$ if it is the head of a rule $p \leftarrow Pre(a), a$ in $L(P)$ such that $a \in A$ and $Pre(a)$ is empty, and $p \in F_{i+1}$ if it is the head of one such rule with $a \in A$ and $Pre(a)$ becoming true at i. For this, it is sufficient

to parallelize the plan π into sets of actions $A_0, \ldots, A_n$, defining F_0 as the set of fluents p true in s and F_{i+1} as the set of fluents that become true after an action in A_i is applied.

Since for each plan π of the problem $P[I = s, G = g]$ we can find a model of the theory $wffc(L(P)) \cup I(s) \cup g$ with the same cost, and vice versa, the cost of the problem is equal to the cost of the theory. □

Proof of Theorem 7. Direct from Theorem 3 and Proposition 6. □

Proof of Proposition 8. Let $n = \langle s, t\rangle$ and $n' = \langle s', t'\rangle$ be two nodes such that n' is the successor of n upon the application of action a. Let Δt be the set $t' - t$ of fluents. Then, monotonicity means

$$h^+_{c_t}\big(P[I = s]\big) \leqslant \left[c(a) + \textstyle\sum_{p\in\Delta t} c(p)\right] + h^+_{c_{t'}}\big(P[I = s']\big).$$

The expression $c(a) + \sum_{p\in\Delta t} c(p)$ stands for the cost of applying action a plus the cost of the atoms $p \in \Delta t$ it introduces. Now, for the state s'' that is the result of applying a on s in the relaxation P^+, the triangle inequality

$$h^+_{c_t}\big(P[I = s]\big) \leqslant \left[c(a) + \textstyle\sum_{p\in\Delta t} c(p)\right] + h^+_{c_{t'}}\big(P[I = s'']\big).$$

must hold due the definition of the heuristic h^+_c and the principle of optimality: the cost of the best plan from s corresponds to the min value of the right-hand side expression over all the actions a applicable in s. At the same time, $s' \subseteq s''$ as s' is s'' with some atoms true in s possibly deleted. It follows then that $h^+_{c_{t'}}(P[I = s'']) \leqslant h^+_{c_{t'}}(P[I = s'])$ as any plan for $P^+([I = s'])$ must be a plan for $P^+([I = s''])$ with no greater cost. The first inequality then follows from the second one. □

Proof of Proposition 9. We need to show that A* with a monotone heuristic and the revised termination condition produces optimal plans even in the presence of negative costs. Monotonicity implies that the evaluation function $f(n) = g(n) + h(n)$ does not decrease along any search path $n_1, n_2, n_3, \ldots$; i.e., $f(n_1) \leqslant f(n_2) \leqslant f(n_2) \leqslant \cdots$, and hence, that no node is expanded twice. (Otherwise, if n_i is expanded along a path with evaluation function $f(n_i) = C$, when there is second path $n_0, n_1, \ldots, n_i$ with strictly less cost to n_i that has not been fully expanded yet, there must be some node n_k along this path in OPEN with $f(n_k) < C$ due to monotonicity, that should have been expanded instead.) Then, since the number of nodes is finite, the revised A* algorithm must eventually terminate with a solution (if the problem has a solution). From the definition of the termination condition, this solution has the best cost among all the solutions found so far, and due to monotonicity, it has cost as good as the best solutions that can be found through the nodes currently in the OPEN list. □

Proof of Theorem 14. The only non-trivial expression to prove is $h^+_c(P, C) \leqslant c^*(P, C)$, as $h^+_c(P) \leqslant h^+_c(P, C)$ is direct, given that C simply pushes the cost of some plans in the relaxation P^+ to ∞, while $c^*(P, C) = c^*(P)$ follows from the validity of C. For proving $h^+_c(P, C) \leqslant c^*(P, C)$, it suffices to note that any plan π for P that complies with the constraints C is also a plan for the relaxation P^+ that complies with C and has the same cost, as the set of fluents $F(\pi)$ that become true as a result of the application of the plan π is the same in P and P^+. □

Proof of Proposition 15. Similar to the proofs of Proposition 6 and Theorem 7. □

Proof of Theorem 16. Direct from Theorem 3 and Proposition 15. □

Proof of Theorem. 17. We show that every assignment encodes a plan in the relaxation that complies with the constraints and has the same cost, and that every plan in the relaxation that complies with the constraint C and has minimum cost, encodes an assignment with the same cost. So the cost of the best assignment is the cost of the best plan in the relaxation that complies with the constraints. The first statement is direct, the actions $map(x, y)$ for x assigned to y, ordered in any way, form a plan for the relaxation that complies with C. For the second, a best plan in P^+ that complies with C must have at least one action $map(x, y)$ for each x (due to the goal), no two actions $map(x, y)$ and $map(x', y)$ for the same y (due to C), and no two actions $map(x, y)$ and $map(x, y')$ for the same x (as a better plan would be obtained by dropping one of the two actions that are assumed to have positive costs). The result is that every x is mapped into a unique y. □

Proof of Theorem 18. We show a correspondence between the TSP tours and the plans for P^+ that comply with the constraints C in the theorem. If the tour is $x_0, x_1, \ldots, x_n, x_f$, the action sequence $move(x_0, x_1), \ldots, move(x_n, x_{n+1})$ with $x_{n+1} = x_f$ is clearly a plan that complies with C which has the same cost. Indeed, the first action is applicable in the initial state, each action $move(x_i, x_{i+1})$ adds the precondition $at(x_{i+1})$ required by following action (the other precondition $not\text{-}visited(x_{i+1})$ is initially true and remains so until the first action $move(x_k, x_{i+1})$ is applied), and they all add the atoms $visited(x)$ in the goal that no action deletes.

In the other direction, we need to show that the edges (x, y) for the actions $move(x, y)$ in a plan π for P^+ that complies with C, form a tour. For this let τ stand for the longest path $\langle x_0, x_1, \ldots, x_m\rangle$ starting at x_0 such that $move(x_i, x_{i+1})$ is in

the plan. This path must be unique as the constraint C prevents the path from splitting into two or more paths. We have to show then that 1) this path contains all x_i, 2) no vertex x_i, for $i = 0, \ldots, m$ appears more than once, and 3) no other *move* actions are in π. For the first, let us assume that there are vertexes not reached by this path, and let $at(x_k)$ be the first atom made true by π for a vertex x_k not in τ. Now, x_k is not x_0 (else it would in τ), and therefore there must be an action $move(x_i, x_k)$ in π with precondition $at(x_i)$ such that x_i is in τ (else $at(x_k)$ would not be the first atom made true by π not in τ). Yet if $x_i \in \tau$, $i < m$ is not possible, as the actions $move(x_i, x_{i+1})$ and $move(x_i, x_k)$ are incompatible with C, while if $i = m$, we could append the vertex x_k to τ in contradiction with τ being the longest such sequence. So there is no such vertex x_k and 1) is proved. For 2), since the path τ is unique and visits all vertices including x_f that has no successors, it cannot include a loop. Last for 3), actions $move(x_i, y)$ for $y \neq x_{i+1}$, cannot be in π as $move(x_i, y)$ and $move(x_i, x_{i+1})$ would contradict C. At the same time, x_f has no successors, and hence the problem includes no actions $move(x_f, y)$. □

Proof of Theorem 19. We show that there is a correspondence between the Minimum (Directed) Spanning Trees rooted at x_0 for the given graph and the best relaxed plans for the delete-relaxation P^+.

The relaxed plan associated with a Directed Spanning Tree rooted at x_0 can be obtained by including the actions $move(x, y)$ for the edges (x, y) in the Spanning Tree, taking the structure of the tree as the partial order among the actions. It is simple to verify that such actions ordered in any way compatible with this partial order renders a plan for P^+: all actions are applicable given the initial situation and the previous actions in the sequence, and all the goals $visited(x_i)$ for all i are achieved at the end.

Likewise, if $G = (V, E)$ is the graph associated with a relaxed plan π for P^+ by setting V to the set of vertices x_i for the atoms $at(x_i)$ achieved by π in P^+ and E to the edges (x_i, x_k) such $move(x_i, x_k)$ is in π, it follows that G is a Directed Spanning Graph rooted at x_0 for the original graph, which includes all of its vertexes, and paths to each one of them from x_0. In addition, if π is an optimal plan for P^+, then G must be actually an Spanning Tree for the original graph, as due to the positive costs of the edges, and hence of the actions, π will not accommodate two actions $move(x_i, x_k)$ and $move(x_j, x_k)$ leading to the same vertex x_k. Thus every vertex in the original graph will be reachable from x_0 by a single path.

Finally, since every directed spanning tree translates into a plan for P^+ of the same cost, and every optimal plan for P^+ translates into a directed spanning tree with the same cost, then min directed spanning trees rooted at x_0 have the same cost as the best plans for P^+. □

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